1.

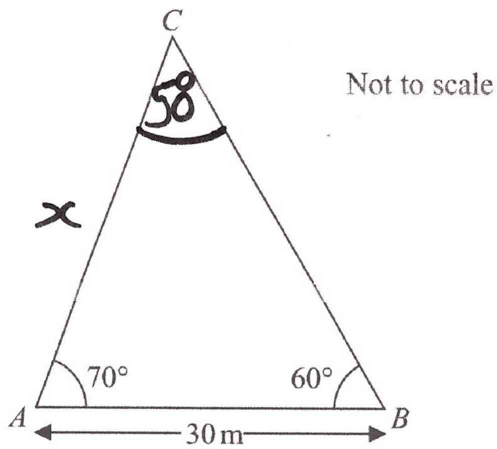

Figure 1

A triangular lawn is modelled by the triangle ABC, shown in Figure 1. The length AB is to be 30 m long.

Given that angle $BAC = 70°$ and angle $ABC = 60°$,

(a) calculate the area of the lawn to 3 significant figures.

(4)

(b) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

(a) Angle $A\hat{C}B = 50°$ (angles in a Δ sum to 180°)

By sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

So, $\frac{x}{\sin 60} = \frac{30}{\sin 50}$

$x = \frac{30}{\sin 50} \times \sin 60$

$x = \frac{30 \sin 60}{\sin 50}$

$x = 33.9\text{m}$ (to 3s.f.)

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Question continued

Area of $\Delta ABC = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(30)(33.9)(\sin 70)$

$= 478.05\ldots$

$= \boxed{478\text{ m}^2 \text{ (to 3s.f.)}}$

(b) It is unlikely to be accurate to the nearest square metre because it is unlikely that the lawn is exactly flat, so modelling by a plane figure may not be accurate.

(Total for Question is 5 marks)

2. In a triangle ABC, side AB has length 10 cm, side AC has length 5 cm, and angle $BAC = \theta$ where θ is measured in degrees. The area of triangle ABC is 15 cm^2

(a) Find the two possible values of $\cos\theta$

(4)

Given that BC is the longest side of the triangle,

(b) find the exact length of BC.

(2)

7a) $\frac{1}{2}(10)(5)\sin\theta = 15$

$\sin\theta = \frac{3}{5}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{16}{25}$

$\cos\theta = \pm\frac{4}{5}$

b) $BC^2 = 10^2 + 5^2 - 2(10)(5)\left(-\frac{4}{5}\right)$

$= 125 + 100\left(\frac{4}{5}\right)$

$= 205$

$BC = \sqrt{205}$

≈ 14.3 cm

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3.

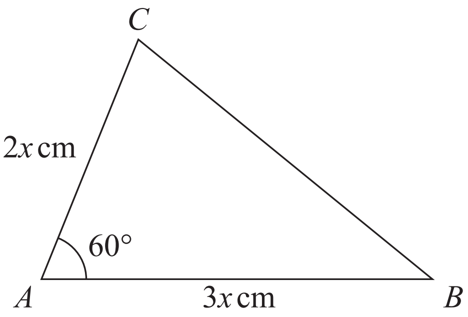

Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) show that $x = 2\sqrt{3}$

(3)

(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(3)

a) $\text{Area} = \frac{1}{2}ab\sin C = 18\sqrt{3}$

$\Rightarrow \frac{1}{2}(2x)(3x)\sin 60 = 18\sqrt{3}$

$\Rightarrow 3x^2 \cdot \frac{\cancel{\sqrt{3}}}{2} = 18\cancel{\sqrt{3}}$

$\Rightarrow x^2 = 12$

$\Rightarrow x = \sqrt{12} = \boxed{2\sqrt{3}}$

b) cosine rule : $a^2 = b^2 + c^2 - 2bc\cos A$

$\Rightarrow BC^2 = (2x)^2 + (3x)^2 - 2(2x)(3x)\cos 60$

$\Rightarrow BC^2 = 13x^2 - 12x^2\left(\frac{1}{2}\right)$

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Question continued

$\Rightarrow BC^2 = 7x^2 = 7(12)$

$\Rightarrow BC = \sqrt{7(12)} = \boxed{2\sqrt{21}}$

(Total for Question is 6 marks)

4.

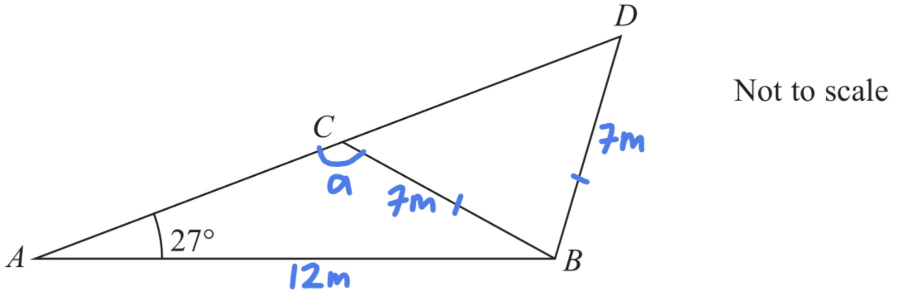


Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, AB, BD, BC and AD.

Given $AB = 12$ m, $BC = BD = 7$m and angle $BAC = 27°$

(a) find, to one decimal place, the size of angle ACB.

(3)

The steel beams can only be bought in whole metre lengths.

(b) Find the minimum length of steel that needs to be bought to make the complete structure.

(3)

a) use sine rule : $\frac{\sin a}{A} = \frac{\sin b}{B}$ ← ratio of sine of angle & opposite side

$\therefore \frac{\sin a}{12} = \frac{\sin 27°}{7}$

$a = \sin^{-1}\left(\frac{12\sin 27°}{7}\right)$

$= 51.1°$ (primary) or $180 - 51.1 = 128.9°$ (secondary)

a is obtuse $\Rightarrow a = 128.9°$

b) use cosine rule to find AD : $a^2 = b^2 + c^2 - 2bc\cos A$

$\measuredangle DCB = 180° - 128.9° = 51.1°$

$\triangle DCB$ is isosceles $\therefore \measuredangle CBD = 180 - 2 \times 51.1° = 77.8°$

Question continued

$\measuredangle CBA = 180^\circ - 27^\circ - 128.9^\circ = 24.1^\circ$

$\therefore \measuredangle ABD = 24.1^\circ + 77.8^\circ = 101.9^\circ$

(check: $\measuredangle ABD = 180^\circ - 27^\circ - 51.1^\circ = 101.9^\circ$)

$\Rightarrow AD^2 = 7^2 + 12^2 - 2 \times 12 \times 7 \cos 101.9^\circ$

$= 227.64$

$\therefore AD = 15.088\text{ m}$

total length $= 12 + 7 + 7 + 15.09$

$= 41.09$

round up to whole metres: 42m bought

(Total for Question is 6 marks)

5. A parallelogram $PQRS$ has area $50\,\text{cm}^2$

Given

- PQ has length 14 cm
- QR has length 7 cm
- angle SPQ is obtuse

find

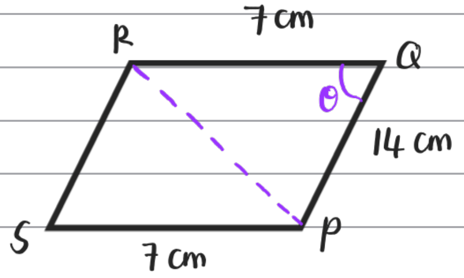
(a) the size of angle SPQ, in degrees, to 2 decimal places,

(3)

(b) the length of the diagonal SQ, in cm, to one decimal place.

(2)

(a)



Area of triangle $= \frac{1}{2} ab \sin C$

$50 = \left(\frac{1}{2}(7)(14)\sin\theta\right) \times 2$ (1)

area of parallelogram = area of triangle × 2

$50 = (49 \sin\theta) \times 2$

$25 = 49 \sin\theta$

$\frac{25}{49} = \sin\theta$

$\theta = 30.67\ldots^\circ$

$\angle SPQ = 180^\circ - 30.67\ldots^\circ$ (1)

$= 149.32\ldots^\circ$

$\approx 149.32^\circ$ (1)

Question continued

(b) Use cosine rule : $C^2 = A^2 + B^2 + 2AB \cos \angle C$

$SQ^2 = 7^2 + 14^2 + 2(7)(14) \cos \angle 149.32°$ 1

$= 413.57...$

$SQ = \sqrt{413.57...}$

$= 20.3$ cm * (1 d.p.) 1

(Total for Question is 5 marks)

6. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A coastguard station O monitors the movements of a small boat.

At 10:00 the boat is at the point $(4\mathbf{i} - 2\mathbf{j})$ km relative to O.

At 12:45 the boat is at the point $(-3\mathbf{i} - 5\mathbf{j})$ km relative to O.

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

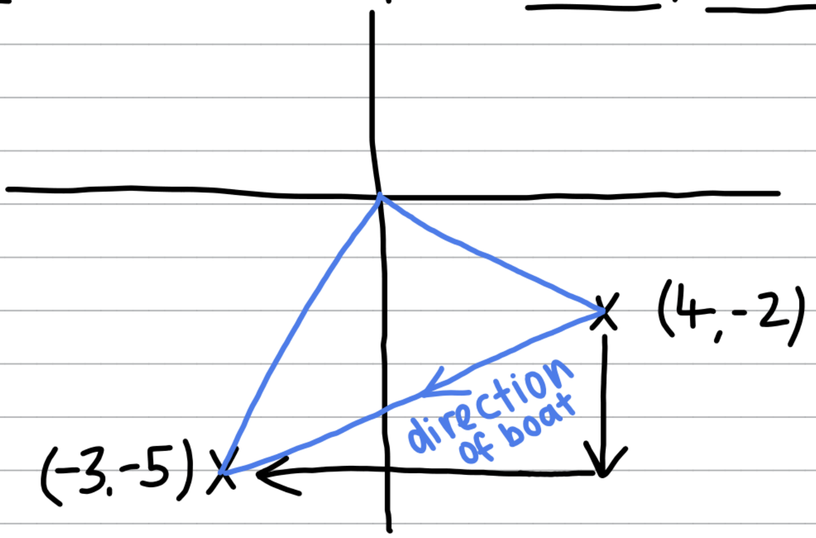
(a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

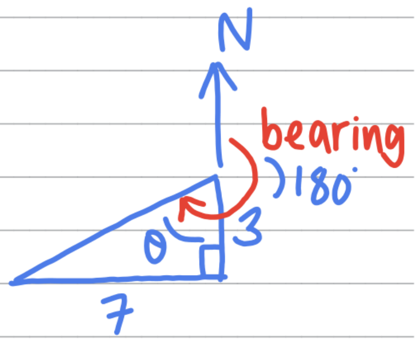
(3)

(b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

a) bearing is measured from north, clockwise





direction vector: $\begin{pmatrix}-3\\-5\end{pmatrix} - \begin{pmatrix}4\\-2\end{pmatrix} = \begin{pmatrix}-7\\-3\end{pmatrix}$

angle needed: $\tan\theta = \dfrac{7}{3}$

$= 66.801°$

total bearing $= 180° + 66.8° = \underline{246.8°}$

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Question continued

b) to find speed, need distance travelled

direction vector $\begin{pmatrix} -7 \\ -3 \end{pmatrix}$ so distance $= \sqrt{7^2 + 3^2} = \sqrt{58}$ km

$\sqrt{(4--3)^2 + (-2+5)^2}$

time: 2hrs45 $\Rightarrow$ 2.75 hours

speed $= \dfrac{\sqrt{58}}{2.75} = 2.77 \text{ kmh}^{-1}$

(Total for Question is 6 marks)