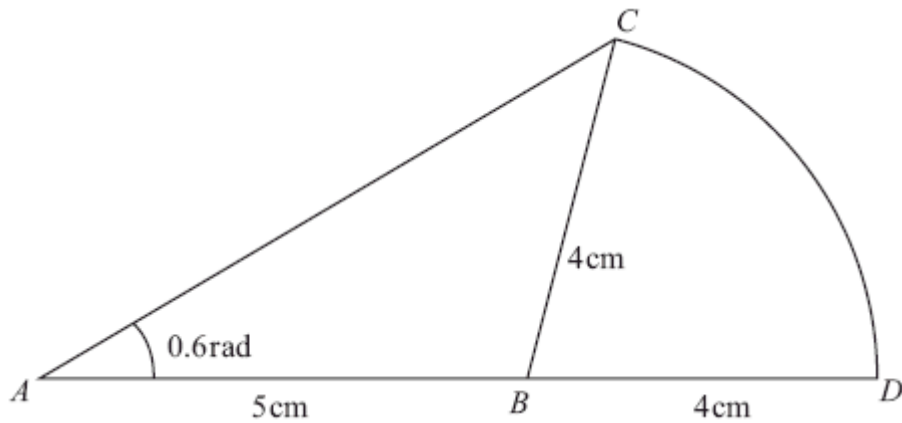


1.



An emblem, as shown in the diagram above, consists of a triangle ABC joined to a sector CBD of a circle with radius 4 cm and centre B . The points A , B and D lie on a straight line with $AB = 5$ cm and $BD = 4$ cm. Angle $BAC = 0.6$ radians and AC is the longest side of the triangle ABC .

(a) Show that angle $ABC = 1.76$ radians, correct to 3 significant figures.

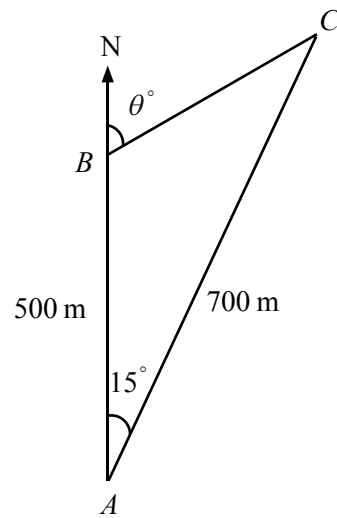
(4)

(b) Find the area of the emblem.

(3)

(Total 7 marks)

2.



The diagram above shows 3 yachts A , B and C which are assumed to be in the same horizontal plane. Yacht B is 500 m due north of yacht A and yacht C is 700 m from A . The bearing of C from A is 015° .

(a) Calculate the distance between yacht B and yacht C , in metres to 3 significant figures.

(3)

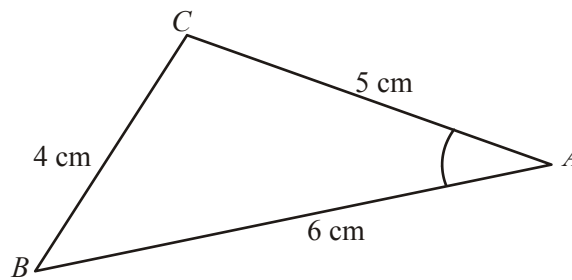
The bearing of yacht C from yacht B is θ° , as shown in the diagram.

(b) Calculate the value of θ .

(4)

(Total 7 marks)

3.



The diagram above shows the triangle ABC , with $AB = 6$ cm, $BC = 4$ cm and $CA = 5$ cm.

(a) Show that $\cos A = \frac{3}{4}$. (3)

(b) Hence, or otherwise, find the exact value of $\sin A$. (2)
(Total 5 marks)

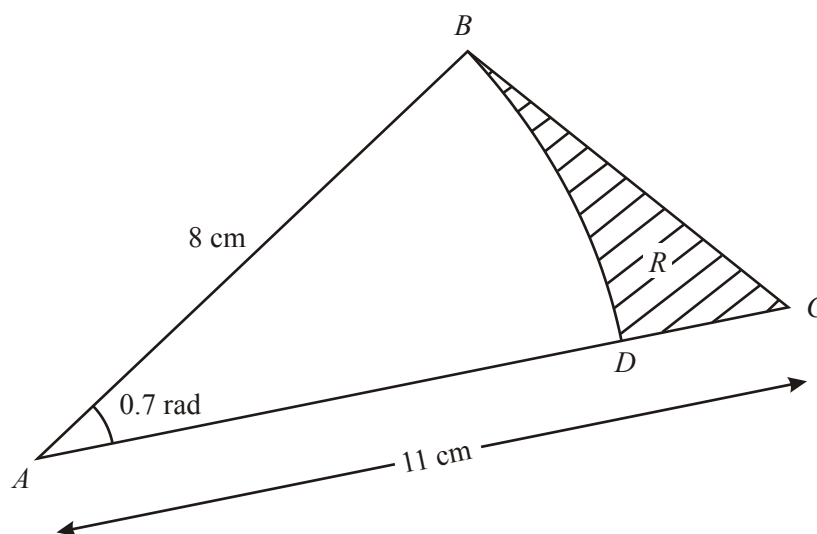
4. In the triangle ABC , $AB = 8$ cm, $AC = 7$ cm, $\angle ABC = 0.5$ radians and $\angle ACB = x$ radians.

(a) Use the sine rule to find the value of $\sin x$, giving your answer to 3 decimal places. (3)

Given that there are two possible values of x ,

(b) find these values of x , giving your answers to 2 decimal places. (3)
(Total 6 marks)

5.



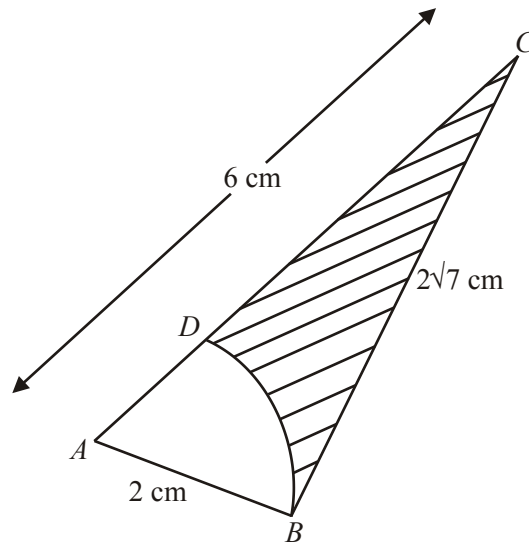
This diagram shows the triangle ABC , with $AB = 8\text{ cm}$, $AC = 11\text{ cm}$ and $\angle BAC = 0.7$ radians. The arc BD , where D lies on AC , is an arc of a circle with centre A and radius 8 cm . The region R , shown shaded in the diagram, is bounded by the straight lines BC and CD and the arc BD .

Find

- (a) the length of the arc BD , (2)
- (b) the perimeter of R , giving your answer to 3 significant figures, (4)
- (c) the area of R , giving your answer to 3 significant figures. (5)

(Total 11 marks)

6.



In $\triangle ABC$, $AB = 2$ cm, $AC = 6$ cm and $BC = 2\sqrt{7}$ cm.

- (a) Use the cosine rule to show that $\angle BAC = \frac{\pi}{3}$ radians.

(3)

The circle with centre A and radius 2 cm intersects AC at the point D , as shown in the diagram above.

Calculate

- (b) the length, in cm, of the arc BD ,

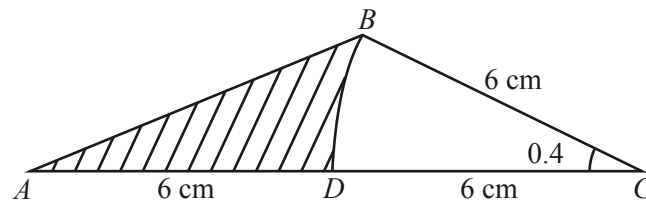
(2)

- (c) the area, in cm^2 , of the shaded region BCD .

(4)

(Total 9 marks)

7.



The diagram above shows a logo ABD .

The logo is formed from triangle ABC . The mid-point of AC is D and $BC = AD = DC = 6$ cm. $\angle BCA = 0.4$ radians. The curve BD is an arc of a circle with centre C and radius 6 cm.

- (a) Write down the length of the arc BD . (1)
- (b) Find the length of AB . (3)
- (c) Write down the perimeter of the logo ABD , giving your answer to 3 significant figures. (1)
- (Total 5 marks)**

1. (a)

Either $\frac{\sin(\hat{A}CB)}{5} = \frac{\sin 0.6}{4}$
 $\therefore \hat{A}CB = \arcsin(0.7058\dots)$
 $= [0.7835\dots \text{ or } 2.358]$
 Use angles of triangle
 $\hat{A}BC = \pi - 0.6 - \hat{A}CB$
 (But as AC is the longest side so)
 $\hat{A}BC = 1.76$ (*) (3sf) [Allow $100.7^\circ \rightarrow 1.76$]
 In degrees $0.6 = 34.377^\circ$, $\hat{A}CB = 44.9$

or $4^2 = b^2 + 5^2 - 2 \times b \times 5 \cos 0.6$
 $\therefore b = \frac{10 \cos 0.6 \pm \sqrt{(100 \cos^2 0.6 - 36)}}{2}$
 $= [6.96 \text{ or } 1.29]$
 Use sine / cosine rule with value for b
 $\sin B = \frac{\sin 0.6}{4} \times b$ or $\cos B = \frac{25 + 16 - b^2}{40}$
 (But as AC is the longest side so)
 $\hat{A}BC = 1.76$ (*) (3sf)

A1

4

Note

- 1st for correct use of sine rule to find ACB or cosine rule to find b (M0 for ABC here or for use of $\sin x$ where x could be ABC)
- 2nd for a correct expression for angle ACB (This mark may be implied by .7835 or by $\arcsin(.7058)$) and needs accuracy. In second method this is for correct expression for b – may be implied by 6.96. [Note $10 \cos 0.6 \approx 8.3$] (do not need two answers)
- 3rd for a correct method to get angle ABC in method (i) or $\sin ABC$ or $\cos ABC$, in method (ii) (If $\sin B > 1$, can have M1A0)

A1 also for correct work leading to 1.76 3sf. Do not need to see angle 0.1835 considered and rejected.

- 1st for a correct expression for sector area or a value in the range 11.0 – 11.1
- 2nd for a correct expression for the area of the triangle or a value of 9.8

Special case

If answer 1.76 is assumed then usual mark is M0 M0 M0 A0.
 A Fully checked method may be worth M0 A0.
 A maximum of 2 marks. The mark is either 2 or 0.

Either for $\hat{A}CB$ is found to be 0,7816 (angles of triangle) then

for checking $\frac{\sin(\hat{A}CB)}{5} = \frac{\sin 0.6}{4}$ with conclusion giving

numerical answers

This gives a maximum mark of 2/4

OR for b is found to be 6.97 (cosine rule)

for checking $\frac{\sin(ABC)}{b} = \frac{\sin 0.6}{4}$ with conclusion giving numerical answers

This gives a maximum mark of 2/4

Candidates making this assumption need a complete method. They cannot earn M1M0.

So the score will be 0 or 2 for part (a). Circular arguments earn 0/4.

(b) $\left[\widehat{CBD} = \pi - 1.76 = 1.38 \right]$ Sector area $= \frac{1}{2} \times 4^2 \times (\pi - 1.76)$
 $= [11.0 \sim 11.1] \frac{1}{2} \times 4^2 \times 79.3$ is M0

Area of $\triangle ABC = \frac{1}{2} \times 5 \times 4 \times \sin(1.76) = [9.8]$ or $\frac{1}{2} \times 5 \times 4 \times \sin 101$ A1 3

Required area = awrt 20.8 or 20.9 or 21.0 or gives 21 (2sf) after correct work.

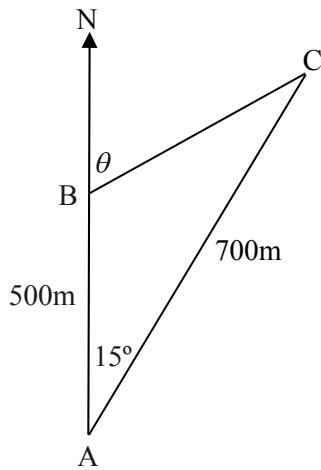
Note

Ignore 0.31 (working in degrees) as subsequent work.

A1 for answers which round to 20.8 or 20.9 or 21.0. No need to see units.

[7]

2.



(a) $BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15^\circ$ M1A1
 (= 63851.92...)
 $BC = 253$ awrt A1 3

If use $\cos 15^\circ = \dots$, then A1 not scored until written as

$BC^2 = \dots$ correctly

Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC

Finding value for BX and CX and using Pythagoras

$BC^2 = (500 \sin 15^\circ)^2 + (700 - 500 \cos 15^\circ)^2$ A1

$BC = 253$ awrt A1

$$(b) \quad \frac{\sin B}{700} = \frac{\sin 15}{\text{candidate's } BC}$$

$\sin B = \sin 15 \times 700 / 253_c = 0.716..$ and giving an **obtuse** B

(134.2°) dep on 1st M

$\theta = 180^\circ - \text{candidate's angle } B$ (Dep. on first M only, B can be acute)

$\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9)

A1 4

[46 needs to be from correct working]

Several alternative methods: (Showing the M marks, 3rd M dep. on first M)

$$(i) \quad \cos B = \frac{500^2 + \text{candidate's } BC^2 - 700^2}{2 \times 500 \times \text{candidate's } BC}$$

$$\text{or } 700^2 = 500^2 + BC_c^2 - 2 \times 500 \times BC_c$$

Finding angle B dep., then as above

(ii) 2 triangle approach, as defined in notes for (a)

$$\tan CBX = \frac{700 - \text{value for } AX}{\text{value for } BX}$$

Finding value for $\angle CBX$ ($\approx 59^\circ$)

dep

$$\theta = [180^\circ - (75^\circ + \text{candidate's } \angle CBX)]$$

(iii) Using sine rule (or cos rule) to find C first:

Correct use of sine or cos rule for C Finding value for C

Either $B = 180^\circ - (15^\circ + \text{candidate's } C)$ or $\theta = (15^\circ + \text{candidate's } C)$

(iv) $700 \cos 15^\circ = 500 + BC \cos \theta$ {first two Ms earned in this case}

Solving for θ , $\theta = 45.8$ (allow 46 or 5.7, 45.8, 45.9) A1

Note: S.C. In main scheme, if θ used in place of B ,

third M gained immediately;

Other two marks likely to be earned, too, for correct value of θ stated.

[7]

3. (a) $4^2 = 5^2 + 6^2 - (2 \times 5 \times 6 \cos \theta)$
 $\cos \theta = \frac{5^2 + 6^2 - 4^2}{2 \times 5 \times 6}$ A1
 $\left(= \frac{45}{60} \right) = \frac{3}{4}$ (*) A1cso 3

M: Is also scored for $5^2 = 4^2 + 6^2 - (2 \times 4 \times 6 \cos \theta)$
 or $6^2 = 5^2 + 4^2 - (2 \times 5 \times 4 \cos \theta)$
 or $\cos \theta = \frac{4^2 + 6^2 - 5^2}{2 \times 4 \times 6}$ or $\cos \theta = \frac{5^2 + 4^2 - 6^2}{2 \times 5 \times 4}$.

1st A: Rearranged correctly and numerically correct (possibly unsimplified), in the form $\cos \theta \dots$ or $60 \cos \theta = 45$ (or equiv. in the form $p \cos \theta = q$).

Alternative (verification):

$$4^2 = 5^2 + 6^2 - \left(2 \times 5 \times 6 \times \frac{3}{4} \right) \quad [$$

Evaluate correctly, at least to $16 = 25 + 36 - 45$ [A1]

Conclusion (perhaps as simple as a tick). [A1cso]

(Just achieving $16 = 16$ is insufficient without at least a tick).

(b) $\sin^2 A + \left(\frac{3}{4} \right)^2 = 1$ (or equiv. Pythag. method)

$$\left(\sin^2 A = \frac{7}{16} \right) \sin A = \frac{1}{4} \sqrt{7} \text{ or equivalent exact form, e.g. } \sqrt{\frac{7}{16}}, \sqrt{0.4375} \text{ A1} \quad 2$$

M: Using a correct method to find an equation in $\sin^2 A$ or $\sin A$ which would give an exact value.

Correct answer without working (or with unclear working or decimals): Still scores both marks.

[5]

4. (a) $\frac{\sin x}{8} = \frac{\sin 0.5}{7}$ or $\frac{8}{\sin x} = \frac{7}{\sin 0.5}$, $\sin x = \frac{8 \sin 0.5}{7}$ A1ft
 $\sin x = 0.548$ A1 3

M: Sine rule attempt (sides/angles possibly the "wrong way round").

A1ft: follow through from sides/angles are the "wrong way round".

- (b) $x = 0.58 (\alpha)$ B1
 (This mark may be earned in (a))
 $\pi - \alpha = 2.56$ A1ft 3

[6]

Too many d.p. given:

Maximum 1 mark penalty in the complete question.
 (Deduct on first occurrence).

5. (a) $r\theta = 8 \times 0.7, = 5.6(\text{cm})$ A1 2
 (b) $BC^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \cos 0.7$
 $\Rightarrow BC = 7.098$ A1
 $\Rightarrow \text{Perimeter} = (a) + (11 - 8) + BC, = 15.7(\text{cm})$ A1cao 4
 (c) $\Delta = \frac{1}{2} absinc = \frac{1}{2} \times 11 \times 8 \times \sin 0.7, = \text{AWRT } 28.3$ A1
 Sector = $\frac{1}{2} r^2 \theta = \frac{1}{2} \times 8^2 \times 0.7$ A1
 Area of R = $28.345 \dots - 22.4 = 5.9455 = 5.95(\text{cm}^2)$ A1 5

[11]

6. (a) $\cos A = \frac{6^2 + 2^2 - (2\sqrt{7})^2}{2 \times 6 \times 2}$ A1
 $\cos A = \frac{1}{2} \quad A = \frac{\pi}{3} \text{ radians } (*)$ A1 3
 (b) $r\theta = \frac{2\pi}{3} \quad (= 2.09)$ (Exact or at least 3 s.f.) A1 2
 (c) Sector ABD: $\frac{1}{2} r^2 \theta = \frac{1}{2} \times 2^2 \times \frac{\pi}{3} \quad \left(= \frac{2\pi}{3} \approx 2.094 \dots \right)$
 Triangle ACB: $\frac{1}{2} \times 2 \times 6 \times \sin \frac{\pi}{3} \quad (= 3\sqrt{3} \approx 5.196 \dots)$
 Triangle – Sector = $3\sqrt{3} - \frac{2\pi}{3} \quad (= 3.10175 \dots)$ A1 4

Allow 3.1 or a.w.r.t. 3.10

[9]

7. (a) Arc $BD = r\theta = 0.4 \times 6 = 2.4$ B1 1
- (b) Cosine Rule: $AB^2 = 6^2 + 12^2 - 2 \times 6 \times 12 \times \cos(0.4^\circ) = 47.36\dots$ A1
 $\therefore AB = 6.88\dots\dots$ A1 3
- (c) Perimeter = $6 + 6.88 + 2.4 = 15.3$ (cm) (3 sig. figs) B1 ft 1

[5]

1. (a) This was a discriminating question, as the method required two stages of solution. Candidates could either find the angle ACB using a correct form of the sine rule, then use angles of a triangle, or they could first find the length AC , then use the sine rule. Finding length AC was complicated (requiring a correct cosine rule and use of a quadratic formula) and the former method was easier. Weaker candidates tried to use Pythagoras, despite the triangle not being right angled, or used the sine rule wrongly and manipulated their answer to give the printed solution. Others assumed the printed answer and attempted verification, but this sometimes resulted in circular arguments and frequently the verification was not conclusive due to the angle being given correct to 3sf. This verification method could earn a maximum of 2 out of 4 marks. Some candidates converted in and out of degrees, often successfully.
- (b) Good candidates found the area of the triangle ABC and the area of the sector BCD and added these to give a correct answer. Weak candidates assumed that the emblem was a sector of radius 9 cm and angle 0.6 radians. Some made errors in their use of formulae and included pi erroneously, or neglected the $\frac{1}{2}$ factor. A few used the wrong angle in their formulae or indeed used the wrong formula, confusing arc length or area of a segment with area of a sector.

2. Although this was accessible to all candidates and marks were gained by the vast majority of candidates, it was a little disappointing to see some of the errors made. The fact that BC could not be the largest side of the triangle did not stop answers of over 700m for BC , for example; a quick check of the working might have found the error. The most common mistake in (a), however, was to evaluate $740000 - 700000\cos 15^\circ$ as $40000\cos 15^\circ$, and so $BC = 197\text{m}$ (3 s.f.) was often seen; it was disappointing to see this error at this level.

In part (b) the most common strategy was to use the sine rule to find angle ABC . For the vast majority of such candidates AC was the largest side of the triangle, but there was a widespread lack of awareness that, therefore, ABC was the largest angle in the triangle. Whilst the good candidate's correctly gave the obtuse angle, the most common answers for angle ABC were 45.7° or 45.8° , which resulted in a maximum of 2 marks being available for this part. Candidates who used the cosine rule to find angle ABC , or who found angle ACB first, were much more successful.

3. The typical response to this question scored full marks in part (a) and no marks in part (b). In part (a) the cosine rule was well known and most candidates managed to manipulate convincingly to achieve the correct (given) value of $A \cos$. A few experienced difficulty in making $A \cos$ the subject of their equation, and $61 - 60 \cos A$ occasionally became $1 \cos A$, but otherwise mistakes were uncommon. In part (b), however, the majority of candidates ignored the requirement for an exact value of $\sin A$. The most popular approach was to simply use a calculator to find A and $\sin A$ (≈ 0.66). A significant number of candidates, having used the cosine rule in part (a), thought that they ought to use the sine rule in part (b) and invariably made no effective progress. Others seemed to assume that the triangle was right-angled. It was pleasing to see good candidates producing correct, concise solutions via $\sin^2 A + \cos^2 A = 1$ or equivalent methods. The identity $\sin A + \cos A = 1$ made the occasional appearance.
4. Most candidates were confident in using the sine rule in part (a), although their diagrams were sometimes labelled with angles in the wrong place. Where the given angle was included between their two given sides, the cosine rule was occasionally used, followed by the sine rule, but a method mark and a follow-through accuracy mark were awarded where possible. Although the value of $\sin x$ was required, many candidates by-passed this demand, losing a mark, and gave only the value of x . Occasionally more fundamental mistakes were seen, including the use of just 0.5 instead of $\sin 0.5$ and the assumption that the given triangle was right-angled.
- It became clear in part (b) that many candidates were not competent, or perhaps not comfortable, in the use of radians. Answers were frequently given in degrees, but were sometimes correctly converted into radians at the end. The second possible value of x was frequently omitted. The method mark for the attempt at the second value was, however, awarded for appropriate work in either radians or degrees. A mark was often lost by failure to round answers to the required degree of accuracy
5. Part (a) was answered very well by almost all the candidates but part (b) caused problems for many. Some assumed the triangle was right angled at B , but many tried to use the cosine rule. Despite the formula being given in the new formula sheet a few candidates misquoted it (having $\sin(0.7)$ instead of $\cos(0.7)$) and the radians caused some confusion with many choosing to use degrees and occasionally forgetting to use the degree formula for sector area in part (c). Of those who had a correct expression for BC , some could not evaluate it correctly and others rounded too soon. Most could identify the 3 lengths required for the perimeter but the final mark required an answer of 15.7 only and was sometimes lost due to previous errors or a failure to round at this final stage. Part (c) was, in the main, handled well and most found the area of the sector correctly but some elaborate, and sometimes incorrect, methods for finding the area of the triangle ABC were used. The intended approach was to use $\frac{1}{2} \times 8 \times 11 \times \sin(0.7)$ but some identified this formula but could not apply it correctly (BC was sometimes used instead of AB for example) and a few forgot to switch the mode on their calculator to evaluate $\sin(0.7)$. Nevertheless there were a number of candidates who scored full marks on this part of the question.

6. About half of the candidates failed to complete part (a) correctly, the main problems being the inability either to quote or to use the cosine rule correctly. For those whose method was correct, the actual calculation was usually well done and the use of radians in the given answer caused very few problems.

Answers for the arc length in part (b) were usually correct, and most candidates were able to quote and use the formula for the area of a sector to gain a mark in part (c). Completely correct solutions in part (c) were not common, however, because most candidates were unable to find the area of triangle ABC . The $\frac{1}{2}ab\sin C$ formula was sometimes wrongly used, while another

popular approach was to attempt $\frac{1}{2}bh$, usually assuming that the triangle was right-angled.

Rounding errors sometimes led to the loss of a mark in otherwise correct solutions.

7. No Report available for this question.