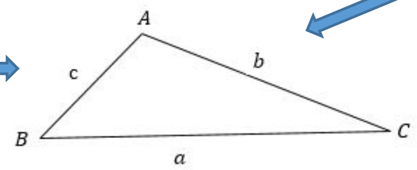
Trigonometric ratios Cheat Sheet

Edexcel Pure Year 1

The cosine rule:
The cosine rule can be used to find missing side and missing angle. The rule can be rearranged in two ways depending on what we need to find, missing side or missing angle

Where a, b, and c are lengths opposite to angles A, B and C respectively.



Find missing side:

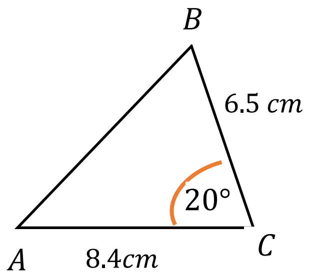
$$a^2 = b^2 + c^2 - 2bc\cos A$$

This version of the rule is used to find a missing side if you know two sides and the angle between them.

Finding missing angle.

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

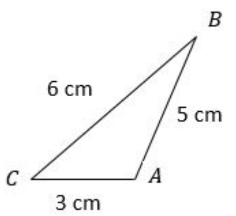
This version of the rule is used to find missing angle given all three sides.
Example 1: calculate the length of the missing side



The missing length is AB which is opposite to angle C.
Use the cosine rule for missing side substitute values of a, b and c
Let $a = 6.5cm$, $b = 8.4\ cm$ and $AB = c = ?$

$c^2 = a^2 + b^2 - 2ab\cos C$
$AB^2 = 6.5^2 + 8.4^2 - 2 \times 6.5 \times 8.4 \times \cos 20° = 10.1955$
$AB = \sqrt{10.1955} \ldots = 3.19cm$

Example 2: Find the size of the smallest angle in a triangle whose side have length 3cm, 5cm and 6cm



Start by drawing the triangle and label it say ABC. The smallest angle is opposite is to the smallest side so angle B is the required angle. Use the cosine rule for missing angle and substitute values of a, b and c.

$a = 6cm, b = 3cm,$ and $c = 5cm$
$b^2 = a^2 + c^2 - 2ac\cos B$

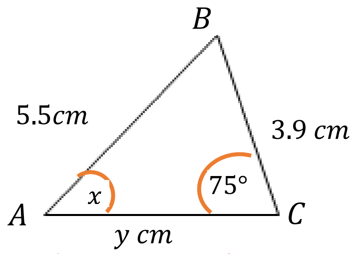
$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{6^2+5^2-3^2}{2\times 6\times 5} = 0.866\ldots\ldots$$

$B = \cos^{-1} 0.8666..$
$B = 29.9°$
Hence, the smallest angle is 29.9°

The sine Rule.
The sine rule can be used to work out missing side or angles in triangles. Similar to cosine rule, sine rule can also be rearranged in two ways to find either missing angle or missing side.
Please refer to the figure shown by arrow for the sine rule.
Where a, b, and c are lengths opposite to angles A, B and C respectively.

Finding missing side:	Finding missing angle
$\frac{\boldsymbol{a}}{\sin \boldsymbol{A}} = \frac{\boldsymbol{b}}{\sin \boldsymbol{B}} = \frac{\boldsymbol{c}}{\sin \boldsymbol{C}}$	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example 3: Work out the values of x and y



In this problem, there is a missing side as well as a missing angle. You will have to use both versions of sine rule.

Finding missing angle:
The side opposite to angle x is length $BC = a = 3.9\ cm$
$a = 3.9\ cm, c = 5.5\ cm, C = 75°, x = ?$
Use the sine rule for missing angle and substitute values of a, c and angle C

$$\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \frac{\sin x}{3.9} = \frac{\sin 75°}{5.5} \Rightarrow \sin x = \frac{3.9 \times \sin 75°}{5.5} = 0.68493$$

$x = \sin^{-1}(0.68493) = 43.23°$ ← Using sine inverse to find x

Finding missing angle:
In order to calculate, we need the angle opposite to length y which is $\angle ABC$
$\angle ABC = 180° - (75 + 43.2)° = 61.8°$
Use the sine rule for missing angle and substitute values of c, angle B and C.

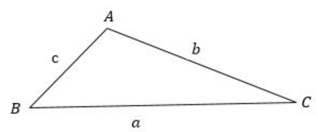
$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{y}{\sin 61.8°} = \frac{5.5}{\sin 75} \Rightarrow y = \frac{5.5 \times \sin 61.8°}{\sin 75} = 5.018$$

$y = 5.02\ cm$

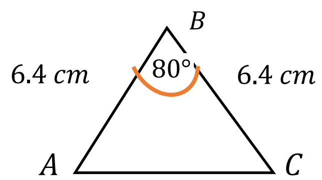
Two solutions for sine:
The sine rule sometimes produces two possible solutions for a missing angle as $\sin\theta = \sin(180° - \theta)$

Areas of triangles:
In this topic you will learn to calculate area of any triangle given 2 sides and the angle between them

$$A = \frac{1}{2}ab\ \sin C$$



Example 4: Calculate the area of triangle.
The angle between two sides AB and BC is angle B
AB is opposite to angle C so AB = c and AC is opposite to angle B so AC = b



Area$= \frac{1}{2}ac\ \sin B$
$A = \frac{1}{2} \times 6.4 \times 6.4\ \times \sin 80°$
$A = 20.16..$
$A = 20.2\ cm^2$

Solving Traingle problems:
Problems involving triangles can be solved by using sine rule, cosine rule along with pythagoras theorem and standard right-angled triangle trigonometry.

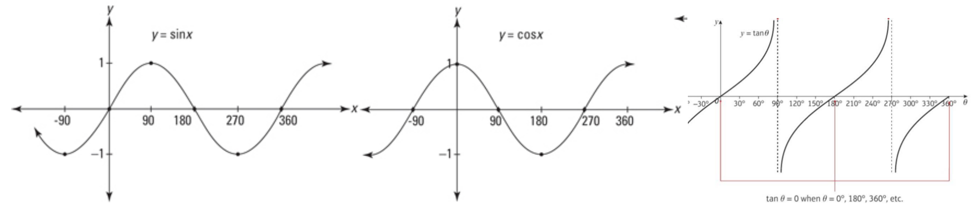
In this section you will learn when to use the above mentioned rules.
Right-angled triangle: Try using basic trigonometry and Pythagoras's theorem to work out other information
Not Right-angled triangle: Use the Sine rule or the Cosine Rule. You can use the rules depending on what information is given.

Use Sine rule	Use Cosine rule
when you are considering 2 angles and 2 sides	when you are considering 3 sides and 1 angle

Graphs of sine, cosine and tangent:
In this section you will have to sketch the graphs of sine, cosine and tangent.
All three graphs are periodic i.e. they repeat themselves after a certain interval.
The below table will help you with properties of the three graphs

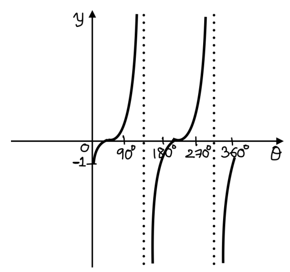
$y = \sin\theta$	$y = \cos\theta$	$y = \tan\theta$
Crosses the x axis at ..,$-\mathbf{180°, 0, 180°, 360°}$,..	Crosses the x axis at ..,$-90°, 90°, 270°, 450°$,..	Crosses the x axis at ..,$-180°, 0, 180°, 360°$,..
Maximum value = 1 Minimum value = -1	Maximum value = 1 Minimum value = -1	No maximum value or minimum value
		Has vertical asymptotes At $x = -90°, 90°, 270°$,..

You can refer to the graphs below for sine, cosine and tangent graphs



Transforming trigonometric graphs:
In chapter 4, you have learned transformations i.e. translation and reflection. In this section you will have to apply the knowledge of transformations in trigonometric functions and sketch the new curve.

Example 5
Sketch the graph of $y = \tan(\theta - 45°)$
The graph of $y = \tan(\theta - 45°)$ is the graph of $\tan\theta$ translated by 45° to the right.
Remember $\boldsymbol{f(x+\theta)} \Rightarrow \boldsymbol{\theta}$ **shifted to LEFT and** $\boldsymbol{f(x-\theta)} \Rightarrow \boldsymbol{\theta}$ **shifted to the RIGHT**



The graphs will shift by 45° to the right
So if $\tan\theta$ meets the θ- axis at$(0°, 0°)$ then $\tan(\theta - 45°)$ meets the θ - axis at $(0° + 45°, 0°) = (45°, 0°)$
Hence,
The graph meets the θ axis at $(45°, 0), (225°, 0)$
And to find, where the graph meets the y-axis do the following
You know that $\theta = 0°$ on y-axis,
So $y = \tan(\theta - 45°) = \tan(0 - 45°) = \tan(-45°) = -1$
Hence the graph meets the y- axis at $(0, -1)$ and has asymptotes at $\theta = 135°$and $\theta = 315°$