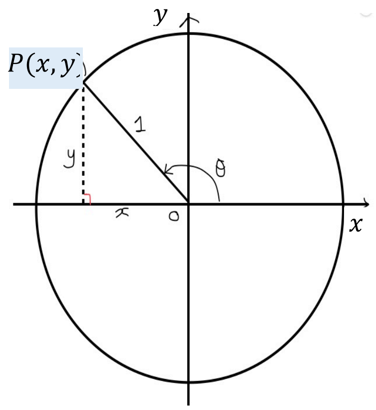
Trigonometric identities Cheat Sheet

Edexcel Pure Year 1

Angles in all four quadrants

Unit circles:

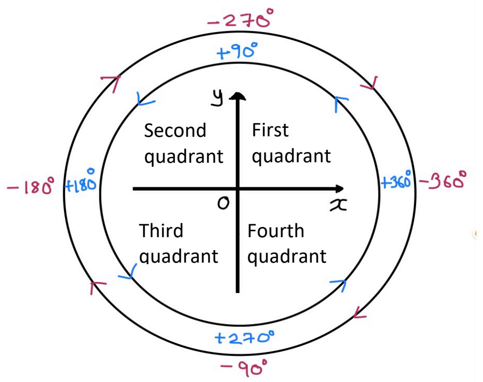
A unit circle is a circle with radius of 1 unit. It will help you understand the trigonometric ratios.



For a point $P(x,y)$ on a unit circle such that OP making an angle with the positive x-axis
$\cos\theta = x$-coordinate of P
$\sin\theta = y$- coordinate of P
$\tan\theta = \frac{y}{x}$ =gradient of OP
You always start measuring θ from positive x-axis
Positive angles ⟺ Anti-clock wise
Negative angles ⟺ Clockwise

With the help of unit circle you can find values and signs of sine, cosine and tangent.

The x-y plane is divided into quadrants:

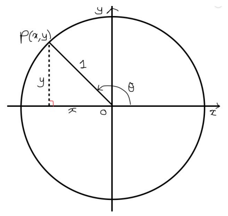


Angles may lie outside the range 0-360°, but they always lie in one of the four quadrants.
For e.g. 520° is equivalent to $520° - 360° = 160°$ which lies in second quadrant

Example 1:

Find the signs of $\sin\theta$, $\cos\theta$ and $\tan\theta$ in the second quadrant.

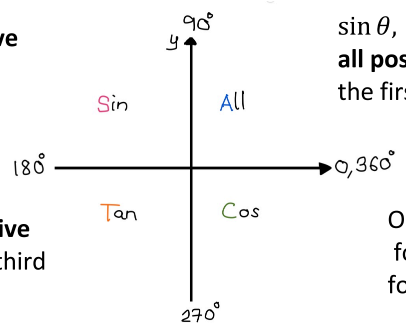
Draw a circle with centre 0 and radius 1, with $P(x,y)$ in the second quadrant.



You know that x is $-ve$ and y is $+ve$ in the second quadrant
$\sin\theta = +ve$, $\cos\theta = -ve$
$\tan\theta = \frac{+ve}{-ve} = -ve$
So, only $\sin\theta$ is $+ve$ in the second quadrant

With the help of the following diagram, you can determine the signs of each of the trigonometric ratios

Only **sin θ is positive** for angle θ in the second quadrant.



$\sin\theta$, $\cos\theta$ and $\tan\theta$ are **all positive** for angle θ in the first quadrant.

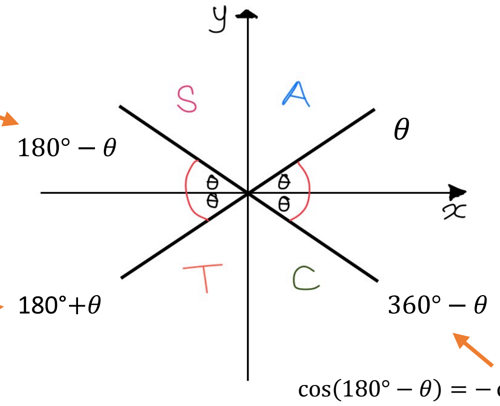
Only **tan θ is positive** for angle θ in the third quadrant

Only **cos θ is positive** for angle θ in the fourth quadrant.

You can use the following rules to find sin, cos or tan of any positive or negative angle using the corresponding acute angle made with the x-axis

$\sin(180° - \theta) = +\sin\theta$
$\sin(180° + \theta) = -\sin\theta$
$\sin(360° - \theta) = -\sin\theta$

$\tan(180° - \theta) = -\tan\theta$
$\tan(180° + \theta) = +\tan\theta$
$\tan(360° - \theta) = -\tan\theta$



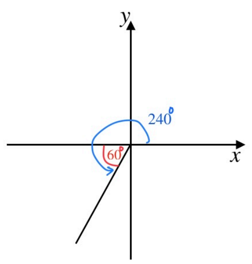
$\cos(180° - \theta) = -\cos\theta$
$\cos(180° + \theta) = -\cos\theta$
$\cos(360° - \theta) = \cos\theta$

Example2:

Express the following in terms of trigonometric ratios of acute angles.

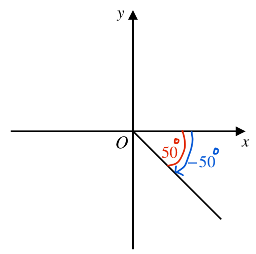
a. $\sin 240°$ b. $\cos(-50°)$

a.



The angle 240° is obtuse and measured from the $+ve$ x -axis anti-clockwise.
So the acute angle is 60°
sin is – ve in the third quadrant
So $\sin 240° = -\sin 60°$

b.

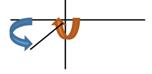


The angle $-50°$ is the angle measured from the positive x- axis clockwise.
50° is the acute angle.
cos is $+ve$ in the fourth quadrant
So $\cos(-50°) = \cos 50°$

Example 3:

Given that θ is an acute angle, express $\tan(\theta - 540°)$in terms of $\tan\theta$

To express $\tan(\theta - 540°)$ in terms of $\tan\theta$, we need to find in which quadrant the angle $\theta - 540°$ lies.
You know that 540° is equivalent to $540° - 360° = 180°$
⇒ $-540°$ is equivalent to $-180°$ ⇒ 180° clockwise and θ = anti-clockwise
So first you will go 180° clockwise and then θ anti-clockwise which will be in the third quadrant.
tan is +ve in the third quadrant
Hence, $\tan(\theta - 540°) = \tan\theta$



Exact values of trigonometric ratios.

You can find exact values of sin, cos and tan of 30°, 45° and 60°. Please refer the table below for the exact values.

	30°	45°	60°
sin θ	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan θ	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Trignometric Identities:

Equation of unit circle is $x^2 + y^2 = 1$
As we know $\cos\theta = x$ and $\sin\theta = y$ ⇒ $\cos^2\theta + \sin^2\theta = 1$
For all values of θ,$\sin^2\theta + \cos^2\theta \equiv 1$
For all values of θ, such that $\cos\theta \neq 0$, $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$
You can use the above identities to simplify trigonometric expressions and complete proofs

Example 4: Simplify $a.\ 5\sin^2 3\theta + 5\cos^2 3\theta$ $b.\ \frac{\sqrt{(1-\cos^2 x)}}{\cos x}$

a. Start by factorising the equation
$\Rightarrow 5(\sin^2 3\theta + \cos^2 3\theta)$
$\Rightarrow 5 \times 1 = 5$ As $\sin^2\theta + \cos^2\theta \equiv 1 \Rightarrow \sin^2 3\theta + \cos^2 3\theta = 1$

b.
$\frac{\sqrt{(1-\cos^2 x)}}{\cos x} = \frac{\sqrt{\sin^2\theta}}{\cos\theta}$ ← As $\cos^2\theta + \sin^2\theta = 1 \Rightarrow (\sin^2\theta = 1 - \cos^2\theta)$

$\Rightarrow \frac{\sqrt{(1-\cos^2 x)}}{\cos x} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

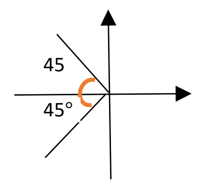
Simple Trignometric equations.

In this section you will learn to solve simple trignometric equations of the form $\sin\theta = k$, $\cos\theta = k$ (where $-1 \le k \le 1$) and $\tan\theta = p$ (where $p \in \mathbb{R}$)
$-1 \le k \le 1$ as sin and cos has maximum $= 1$ and minimum $= -1$
$p \in \mathbb{R}$ as tan has no maximum or minimum value

Example 5: Solve the equation $2\cos\theta = -\sqrt{2}$ for θ, in the interval $0 \le x \le 360°$

First rearrange the equation in the form $\cos\theta = k$
So $\cos\theta = \frac{-\sqrt{2}}{2} = -0.7071$
$\theta = \cos^{-1}(-0.7071) = 45°$

The values you get on calculator taking inverse of trigonometric functions are called principal values. But principal values will not always be a solution to the equation.



As $\cos\theta = -0.7071\ and\ \theta = 45° \Rightarrow$ cos is negative so you need to look θ in the 2nd and 4th quadrant
45° is the acute angle (i.e angle made with the horizontal axis) but we are looking for the angle made from the positive x- axis anti-clockwise.
So, there are two solutions
$180° - 45° = 135°$ and $180° + 45° = 225°$
Hence, θ= 135° or θ= 225°

Harder trigonometric equations:

You will have to solve equations of the form
$\sin n\theta = k, \cos n\theta = k\ and\ \tan n\theta = p$
$\sin(\theta + \alpha) = k, \cos(\theta + \alpha) = k\ and\ \tan(\theta + \alpha) = p$
It is same as solving simple equations, but will have some extra steps

Example 6: Solve the equation $\sin(x + 60°) = 0.3$ in the interval $0 \le x \le 360°$

Let $X = x + 60° \Rightarrow \sin X = 0.3$
The interval for X will be $0 + 60° \le X \le 360° + 60° \Rightarrow 60° \le X \le 420°$
$X = \sin^{-1} 0.3 = 17.45°$, principal value
Sin is positive which mean 17.45° should be in the 1st and 2nd quadrant.
One of the solution will be $180° - 17.45° = 162.54°$
Now the other solution could be 17.45° but $60° \le X \le 420°$, so it cannot be 17.45°.
So start from $+ve$ x-axis and measure one full circle i.e. 360° and add 17.5°
$\Rightarrow 360° + 17.45° = 377.45°$ So $X = 162.54\ldots°, 377.45\ldots°$
Subtract 60° from each value: Hence, $x = 102.5°\ or\ 317.5°$

Equations and Identities:

You will have to solve quadratics equations in $\sin\theta, \cos\theta\ and\ \tan\theta$

Example 7: Solve for θ, in the interval $0 \le x \le 360°$, the equation $2\cos^2\theta - \cos\theta - 1 = 0$

Start by factorising the equation as you do for quadratic equation
$2\cos^2\theta - \cos\theta - 1 = 0$ Compare with $2x^2 - x - 1 = (2x+1)(x-1)$
So $(2\cos\theta + 1)(\cos\theta - 1) = 0$
$\cos\theta = -\frac{1}{2}\ or\ \cos\theta = 1$ Set each factor equal to 0 thereby finding two sets of solutions

$\cos\theta = -\frac{1}{2} \Rightarrow \theta = 60°$
Cosine is negative implies solution is in the 2nd and 3rd quadrants
In the 2nd quadrant $\theta = 180 - 60 = 120°$. So, one solution is 120°
In the 3rd quadrant $\theta = 180 + 60 = 240°$.
So, the other solution in the 3rd quadrant will be 240°
$\cos\theta = 1\ so\ \theta = 0\ or\ 360°$
So the solutions are
$\theta = 0°, 120°, 240°, 360°$