SEQUENCES AND SERIES — *Answers*

1 **a** $a + d = 40$, $a + 4d = 121$
subtracting, $3d = 81$
$d = 27$
sub. $a = 13$
b $S_{25} = \frac{25}{2}[26 + (24 \times 27)] = 8425$

2 **a** 3, 7, 11, 15, 19
b AP: $a = 3$, $d = 4$, $n = 20$
$S_{20} = \frac{20}{2}[6 + (19 \times 4)]$
$= 820$

3 **a** $(2t - 5) - t = 8.6 - (2t - 5)$
$t = 6.2$
b $u_1 = 6.2$, $u_2 = 12.4 - 5 = 7.4$
$a = 6.2$, $d = 7.4 - 6.2 = 1.2$
$u_{16} = 6.2 + (15 \times 1.2) = 24.2$
c $S_{20} = \frac{20}{2}[12.4 + (19 \times 1.2)] = 352$

4 **a** $S_n = \frac{1}{2}n(n + 1)$
b $= S_{400} - S_{199}$
$= \frac{1}{2} \times 400 \times 401 - \frac{1}{2} \times 199 \times 200$
$= 80\,200 - 19\,900 = 60\,300$
c $\frac{1}{2}N(N + 1) = 4950$
$N^2 + N - 9900 = 0$
$(N + 100)(N - 99) = 0$
$N > 0 \;\therefore\; N = 99$

5 **a** $u_2 = k + 1$
$u_3 = k + (k + 1)^2 = k^2 + 3k + 1$
b $k^2 + 3k + 1 = 1$
$k(k + 3) = 0$
$k \neq 0 \;\therefore\; k = -3$
c $u_{25} = 1$
$u_1 = 1 \;\Rightarrow\; u_3 = 1$
$\therefore\; u_n = 1$ for all odd values of n

6 **a** AP: $a = 3$, $d = 3$
$500 \div 3 = 166\tfrac{2}{3} \;\therefore\; n = 166$
$S_{166} = \frac{166}{2}[6 + (165 \times 3)]$
$= 41\,583$
b AP: $a = 14$, $l = 99$, $n = 18$
$S_{18} = \frac{18}{2}(14 + 99)$
$= 1017$

7 **a** $S_n = a + (a + d) + (a + 2d) + \ldots + [a + (n - 2)d] + [a + (n - 1)d]$
write in reverse
$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + (a + 2d) + (a + d) + a$
adding $2S_n = n \times \{a + [a + (n - 1)d]\}$
$S_n = \frac{1}{2}n[2a + (n - 1)d]$
b $S_{26} = \frac{26}{2}[-2 + (25 \times 6)] = 1924$
$S_{27} = \frac{27}{2}[-2 + (26 \times 6)] = 2079$
$\therefore$ largest $n = 26$

8 $t_2 = 4 + 2k$
$t_3 = 4 - k(4 + 2k)$
$\therefore\; 4 - 4k - 2k^2 = 3$
$2k^2 + 4k - 1 = 0$
$k = \dfrac{-4 \pm \sqrt{16 + 8}}{4} = \dfrac{-4 \pm 2\sqrt{6}}{4}$
$k > 0 \;\therefore\; k = -1 + \frac{1}{2}\sqrt{6}$

9 **a** $= 6 + (19 \times 3) = 63$
b $S_n = \frac{n}{2}[12 + 3(n - 1)] = 270$
$\therefore\; n(3n + 9) = 540$
$n^2 + 3n - 180 = 0$
$(n + 15)(n - 12) = 0$
$n > 0 \;\therefore\; n = 12$

10 **a** $= 3 \times 570 = 1710$
b $= 570 + (2 \times 30) = 630$
c $= 570 + (\frac{1}{2} \times 30 \times 31) = 1035$

11 **a** 2 years = 8×3 months
total = $3 \times S_8$ [AP: $a = 40$, $d = 2$]
$= 3 \times \frac{8}{2}[80 + (7 \times 2)]$
$= 3 \times 376 = £1128$

b n years = $4n \times 3$ months
total = $3 \times S_{4n}$
$= 3 \times \frac{4n}{2}\{80 + [(4n - 1) \times 2]\}$
$= 6n(80 + 8n - 2)$
$= 12n(4n + 39)$

12 AP: $a = 80$, $d = -3$, $n = 45$
$S_{45} = \frac{45}{2}[160 + (44 \times -3)] = 630$

13 **a** $a + 2d = 298$, $a + 7d = 263$
subtracting, $5d = -35$
$d = -7$

b sub. $a = 312$
$312 - 7(n - 1) > 0$
$n < \frac{319}{7}$ $\therefore$ 45 positive terms

c max S_n when $n = 45$
$S_{45} = \frac{45}{2}[624 + (44 \times -7)] = 7110$

14 **a** AP: $a = 10$, $d = 6$
$S_n = \frac{n}{2}[20 + 6(n - 1)]$
$= n(3n + 7)$

b $S_{2n} = 2n[(3 \times 2n) + 7]$
$= 12n^2 + 14n$
required sum = $S_{2n} - S_n$
$= (12n^2 + 14n) - (3n^2 + 7n)$
$= 9n^2 + 7n = n(9n + 7)$

15 **a** $u_2 = k^2 - 2$, $u_4 = k^4 - 4$
$\therefore$ $k^2 - 2 + k^4 - 4 = 6$
$k^4 + k^2 - 12 = 0$
$(k^2 + 4)(k^2 - 3) = 0$
$k^2 = -4$ [no solutions] or 3
$k > 0$ $\therefore$ $k = \sqrt{3}$

b $u_1 = \sqrt{3} - 1$
$u_3 = (\sqrt{3})^3 - 3 = 3(\sqrt{3} - 1) = 3u_1$

16 **a** $(4k - 2) - (k + 4) = (k^2 - 2) - (4k - 2)$
$3k - 6 = k^2 - 4k$
$k^2 - 7k + 6 = 0$

b $(k - 1)(k - 6) = 0$
$k = 1$ or 6
$d = 3k - 6$
$d > 0$ $\therefore$ $k = 6$
$a = 10$, $d = 12$
$u_{15} = 10 + (14 \times 12) = 178$