SEQUENCES AND SERIES — Answers

1 **a** $r = 20\frac{1}{4} \div 27 = \frac{3}{4}$

$a \times (\frac{3}{4})^2 = 27$

$a = \frac{16}{9} \times 27 = 48$

b $S_\infty = \frac{48}{1-\frac{3}{4}} = 192$

2 **a** $\frac{k+4}{k-8} = \frac{3k+2}{k+4}$

$(k + 4)^2 = (3k + 2)(k - 8)$

$k^2 - 15k - 16 = 0$

$(k + 1)(k - 16) = 0$

$k > 0 \;\; \therefore \; k = 16$

b $u_1 = 8,\ u_2 = 20 \;\; \therefore \; a = 8,\ r = \frac{5}{2}$

$u_6 = 8 \times (\frac{5}{2})^5 = 781\frac{1}{4}$

c $S_{10} = \frac{8[(\frac{5}{2})^{10} - 1]}{\frac{5}{2} - 1} = 50\,857.3$

3 **a** $ar = 75,\ ar^4 = 129.6$

$r^3 = 129.6 \div 75 = 1.728$

$r = \sqrt[3]{1.728} = 1.2$

$a = 75 \div 1.2 = 62.5$

b $u_{10} = 62.5 \times (1.2)^9 = 322.5$

c $S_{12} = \frac{62.5[(1.2)^{12} - 1]}{1.2 - 1} = 2473.8$

4 **a** $S_n = a + ar + ar^2 + \ldots ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + \ldots ar^{n-1} + ar^n$

subtracting,

$S_n - rS_n = a - ar^n$

$(1 - r)S_n = a(1 - r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

b $\frac{2[1-(\sqrt{2})^n]}{1-\sqrt{2}} = 126(\sqrt{2} + 1)$

$1 - (\sqrt{2})^n = 63(\sqrt{2} + 1)(1 - \sqrt{2})$

$1 - (\sqrt{2})^n = 63(1 - 2)$

$(\sqrt{2})^n = 64$

$2^{\frac{1}{2}n} = 2^6$

$n = 12$

5 **a** $\frac{18}{1-r} = 15$

$\therefore \; 1 - r = \frac{18}{15} = 1.2$

$r = -0.2$

b $u_3 = 18 \times (-0.2)^2 = 0.72$

c $S_8 = \frac{18[1-(-0.2)^8]}{1-(-0.2)} = 14.9999616$

$S_\infty - S_8 = 0.000\,0384$

6 **a** $S_3 = 5(3^3 - 1) = 130$

$S_2 = 5(3^2 - 1) = 40$

$u_3 = S_3 - S_2 = 90$

b $S_{n-1} = 5(3^{n-1} - 1)$

$u_n = S_n - S_{n-1} = 5(3^n - 1) - 5(3^{n-1} - 1)$

$= 5[3^n - 3^{n-1}] = 5(3^n)[1 - \frac{1}{3}] = \frac{10}{3}(3^n)$

7 **a** $4 \times (1.25)^7 = 19.1$ mm (3sf)

b GP: $a = 4,\ r = 1.25$

$S_{20} = \frac{4[(1.25)^{20} - 1]}{1.25 - 1} = 1371.8$ mm

$\therefore$ length = 1.37 m (3sf)

8 **a** $ar = 30,\ ar^3 = 2.7 \;\; \therefore \; r^2 = 2.7 \div 30 = 0.09$

$r > 0 \;\; \therefore \; r = \sqrt{0.09} = 0.3$

$a = 30 \div 0.3 = 100$

b $S_\infty = \frac{100}{1-0.3} = 142.9$ (1dp)

9 **a** GP: $a = 27$, $r = 3$

$S_8 = \frac{27(3^8 - 1)}{3 - 1} = 88\,560$

b $\sum_{r=1}^{15} 2^r$: GP, $a = 2$, $r = 2$

$S_{15} = \frac{2(2^{15} - 1)}{2 - 1} = 65\,534$

$\sum_{r=1}^{15} 12r$: AP, $a = 12$, $d = 12$

$S_{15} = \frac{15}{2}[24 + (14 \times 12)] = 1440$

$\sum_{r=1}^{15} (2^r - 12r) = 65\,534 - 1440 = 64\,094$

10 **a** $a = 64$, $ar^2 - ar = 20$

$\therefore\ 64r^2 - 64r = 20$

$16r^2 - 16r - 5 = 0$

b $(4r + 1)(4r - 5) = 0$

$r = -\frac{1}{4}$ or $\frac{5}{4}$

c $r = -\frac{1}{4} \Rightarrow u_4 = 64 \times (-\frac{1}{4})^3 = -1$

$r = \frac{5}{4} \Rightarrow u_4 = 64 \times (\frac{5}{4})^3 = 125$

d $r = -\frac{1}{4} \Rightarrow S_\infty = \frac{64}{1 - (-\frac{1}{4})} = 51\frac{1}{5}$

11 **a** $u_8 = 4 \times (\frac{1}{2})^7 = \frac{1}{32}$

b $u_n = 4 \times (\frac{1}{2})^{n-1}$

$= 2^2 \times 2^{1-n}$

$= 2^{3-n}$

c $S_n = \frac{4[1 - (\frac{1}{2})^n]}{1 - \frac{1}{2}}$

$= 8(1 - 2^{-n})$

$= 8 - (2^3 \times 2^{-n})$

$= 8 - 2^{3-n}$

12 **a** $u_6 = 4 \times 3^6 = 2916$

b GP: $a = 12$, $r = 3$

$S_t = \frac{12(3^t - 1)}{3 - 1} = 6(3^t - 1)$

$\therefore\ 6(3^t - 1) > 10^{25}$

$3^t > \frac{10^{25}}{6} + 1$

$t \lg 3 > \lg(\frac{10^{25}}{6} + 1)$

$t > \frac{\lg(\frac{10^{25}}{6} + 1)}{\lg 3}$

$t > 50.8$ $\therefore$ smallest $t = 51$

13 **a** $a + ar^2 = a(1 + r^2) = 150$

$ar + ar^3 = ar(1 + r^2) = -75$

$\therefore\ r = -75 \div 150 = -\frac{1}{2}$

$a = 150 \div \frac{5}{4} = 120$

b $S_\infty = \frac{120}{1 - (-\frac{1}{2})} = 80$

14 **a** $b - a = (3a + 4) - b$

$2b = 4a + 4$

$b = 2a + 2$

b $\frac{2a + 2}{a} = \frac{6a + 1}{2a + 2}$

$(2a + 2)^2 = a(6a + 1)$

$2a^2 - 7a - 4 = 0$

$(2a + 1)(a - 4) = 0$

a integer $\therefore\ a = 4$

sub. $b = 10$

15 **a** after 4th bounce,

reaches $3 \times (0.6)^4 = 0.3888$ m

b total distance

$= h + 2[0.6h + (0.6)^2h + (0.6)^3h + \dots]$

$= h + 2 \times S_\infty$ of GP, $a = 0.6h$, $r = 0.6$

$= h + \frac{2 \times 0.6h}{1 - 0.6}$

$= h + 3h = 4h$ metres