SEQUENCES AND SERIES — *Answers*

1 **a** $a + 2d = -10$ (1)

$\frac{8}{2}(2a + 7d) = 16 \Rightarrow 2a + 7d = 4$

$2 \times (1) \Rightarrow 2a + 4d = -20$

subtracting, $3d = 24$

$d = 8$

sub. $a = -26$

b $-26 + 8(n - 1) > 300$

$n > 41\frac{3}{4}$ $\therefore$ smallest $n = 42$

2 **a** $a + 2d = \frac{5}{6}$

$a + 6d = 2\frac{1}{3}$

subtracting, $4d = 1\frac{1}{2}$

$d = \frac{3}{8}$

sub. $a = \frac{1}{12}$

b $S_n = \frac{n}{2}[\frac{1}{6} + \frac{3}{8}(n - 1)]$

$= \frac{1}{48}n[4 + 9(n - 1)]$

$= \frac{1}{48}n(9n - 5)$ $[\ k = \frac{1}{48}\ ]$

3 **a** $\frac{9}{2}(2a + 8d) = 126$

$9(a + 4d) = 126$

$a + 4d = 14$

b $\frac{15}{2}(2a + 14d) = 277.5$

$a + 7d = 18.5$

subtracting, $3d = 4.5$

$d = 1.5$

sub. $a = 8$

c $S_{32} = \frac{32}{2}[16 + (31 \times 1.5)] = 1000$

4 **a** $(5k + 3) - (7k - 1) = (4k + 1) - (5k + 3)$

$-2k + 4 = -k - 2$

$k = 6$

b given terms = 41, 33, 25

$d = -8$

smallest +ve term = $25 + (3 \times -8) = 1$

c consider series of +ve terms in reverse

$a = 1, d = 8$

$S_r = \frac{r}{2}[2 + 8(r - 1)] = r(4r - 3)$

5 **a** AP: $a = 4$, $l = 120$, $n = 30$

$S_{30} = \frac{30}{2}(4 + 120) = 1860$

b **i** $= \sum_{r=1}^{30} 4r + 30 = 1890$

ii $= 2 \times \sum_{r=1}^{30} 4r - (30 \times 5)$

$= (2 \times 1860) - 150 = 3570$

6 **a** $500 + (7 \times 40) = £780$

b AP: $a = 500$, $d = 40$

$S_n = \frac{n}{2}[1000 + 40(n - 1)] = 20n(n + 24)$

c AP: $a = 400$, $d = 60$

$S_n = \frac{n}{2}[800 + 60(n - 1)] = 10n(3n + 37)$

$\therefore$ $20n(n + 24) = 10n(3n + 37)$

$n \neq 0$ $\therefore$ $2(n + 24) = (3n + 37)$

$n = 11$ $\therefore$ 11 years

7 **a** $S_n = 2 + 4 + 6 + \ldots + (2n - 2) + 2n$

write in reverse

$S_n = 2n + (2n - 2) + \ldots + 6 + 4 + 2$

adding, $2S_n = n \times (2n + 2)$

$S_n = n(n + 1)$

b integers 200 to 800, AP: $n = 601$

$S_{601} = \frac{601}{2}(200 + 800) = 300\,500$

integers 200 to 800 divisible by 4

AP: $a = 200$, $l = 800$

$200 + 4(n - 1) = 800 \Rightarrow n = 151$

$S_{151} = \frac{151}{2}(200 + 800) = 75\,500$

required sum = $300\,500 - 75\,500$

$= 225\,000$

8 **a** $S_n = \frac{1}{2}n[2a + (n - 1)d]$

b $S_2 = \frac{2}{2}(2a + d) = 2a + d$

$S_6 = \frac{6}{2}(2a + 5d) = 6a + 15d$

$S_8 = \frac{8}{2}(2a + 7d) = 8a + 28d$

$2(S_6 - S_2) = 2[(6a + 15d) - (2a + d)]$

$= 2(4a + 14d)$

$= 8a + 28d = S_8$

c for +ve terms $40 - 3(n - 1) > 0$

$n < \frac{43}{3}$ $\therefore$ 14 terms

$\therefore$ $S_{14} = \frac{14}{2}[80 + (13 \times -3)] = 287$

9 **a** **i** $u_4 - u_1 = x + 3$

$u_7 = u_4 + (x + 3) = 3x + 6$

ii $3d = x + 3$

$d = \frac{1}{3}x + 1$

iii $S_{10} = \frac{10}{2}[2x + 9(\frac{1}{3}x + 1)]$

$= 5[2x + 3x + 9] = 25x + 45$

b $x + 19(\frac{1}{3}x + 1) = 52$

$3x + 19x + 57 = 156$

$x = \frac{99}{22} = \frac{9}{2}$ or $4\frac{1}{2}$

10 $S_{20} = \frac{20}{2}(2a + 19d) = 20a + 190d$

$S_{30} = \frac{30}{2}(2a + 29d) = 30a + 435d$

$S_{30} - S_{20} = 10a + 245d$

$\therefore$ $20a + 190d = 10a + 245d$

$10a = 55d$

$2a = 11d$

$\therefore$ $a : d = 11 : 2$

11 **a** $S_6 = 12(16 - 6) = 120$

$S_5 = 10(16 - 5) = 110$

$u_6 = S_6 - S_5 = 10$

b $S_n = 2n(16 - n) = 32n - 2n^2$

$S_{n-1} = 2(n - 1)[16 - (n - 1)]$

$= 2(n - 1)(17 - n)$

$= -2n^2 + 36n - 34$

$u_n = S_n - S_{n-1}$

$= (32n - 2n^2) - (-2n^2 + 36n - 34)$

$= 34 - 4n$

c $u_{n-1} = 34 - 4(n - 1) = 38 - 4n$

$u_n - u_{n-1} = (34 - 4n) - (38 - 4n) = -4$

$u_n - u_{n-1}$ constant $\therefore$ arithmetic series

12 **a** **i** $2400 + (5 \times 250) = 3650$

ii AP: $a = 2400$, $d = 250$

$S_{10} = \frac{10}{2}[4800 + (9 \times 250)]$

$= 35\,250$

b AP: $a = 2400$, $d = C$

$\frac{10}{2}[4800 + (9 \times C)] = 40\,000$

$C = \frac{3200}{9} = 356$ (nearest unit)

13 **a** let common difference be d

$S_r = a + (a + d) + (a + 2d) + \ldots + (l - 2d) + (l - d) + l$

write in reverse

$S_r = l + (l - d) + (l - 2d) + \ldots + (a + 2d) + (a + d) + a$

adding, $2S_r = r \times (a + l)$

$S_r = \frac{1}{2}r(a + l)$

b $n = 18$, $l = 68$, $S_{18} = 153$

$\therefore$ $153 = \frac{18}{2}(a + 68)$

$a = 17 - 68 = -51$