

SEQUENCES AND SERIES

Answers

- 1** **a** $d = 6$
 $u_{40} = 4 + (39 \times 6) = 238$ **b** $d = -3$
 $u_{40} = 30 + (39 \times -3) = -87$ **c** $d = 2.3$
 $u_{40} = 8.9 + (39 \times 2.3) = 98.6$
- 2** **a** $a = 7, d = 2$
 $u_n = 7 + 2(n - 1) = 5 + 2n$ **b** $a = \frac{1}{6}, d = \frac{4}{3}$
 $u_n = \frac{1}{6} + \frac{4}{3}(n - 1) = -\frac{7}{6} + \frac{4}{3}n$ **c** $a = 17, d = -8$
 $u_n = 17 - 8(n - 1) = 25 - 8n$
- 3** **a** $a = 8, d = 4, n = 30$
 $S_{30} = \frac{30}{2} [16 + (29 \times 4)]$
 $= 1980$ **b** $a = 60, d = -7, n = 30$
 $S_{30} = \frac{30}{2} [120 + (29 \times -7)]$
 $= -1245$ **c** $a = 7\frac{1}{4}, d = 1\frac{1}{2}, n = 30$
 $S_{30} = \frac{30}{2} [14\frac{1}{2} + (29 \times 1\frac{1}{2})]$
 $= 870$
- 4** **a** $S_{20} = \frac{20}{2} (60 + 136)$
 $= 1960$ **b** $S_{32} = \frac{32}{2} (100 + 84.5)$
 $= 2952$ **c** $S_{17} = \frac{17}{2} [28 + (-20)]$
 $= 68$
- 5** **a** $S_{48} = \frac{48}{2} [4 + (47 \times 9)]$
 $= 10\,248$ **b** $S_{36} = \frac{36}{2} [200 + (35 \times -5)]$
 $= 450$ **c** $S_{55} = \frac{55}{2} [38 + (54 \times 13)]$
 $= 20\,350$
- 6** **a** $8 + 3(n - 1) = 65$
 $n = 20$
 $S_{20} = \frac{20}{2} (8 + 65)$
 $= 730$ **b** $3.4 + 1.2(n - 1) = 23.8$
 $n = 18$
 $S_{18} = \frac{18}{2} (3.4 + 23.8)$
 $= 244.8$ **c** $22 - 8(n - 1) = -226$
 $n = 32$
 $S_{32} = \frac{32}{2} [22 + (-226)]$
 $= -3264$
- 7** **a** $a = 21$
 $21 + 2d = 27$
 $\therefore d = 3$
b $S_{40} = \frac{40}{2} [42 + (39 \times 3)] = 3180$
- 8** $n = 1$, first term $= 7 + 16 = 23$
 $d = 7$
 $S_{35} = \frac{35}{2} [46 + (34 \times 7)] = 4970$
- 9** **a** $a + d = 13$
 $a + 4d = 46$
b subtracting, $3d = 33$
 $d = 11$
sub. $a = 2$
c $u_{40} = 2 + (39 \times 11) = 431$
- 10** **a** $a + 2d = 72$
 $a + 7d = 37$
subtracting, $5d = -35$
 $d = -7$
sub. $a = 86$
b $S_{25} = \frac{25}{2} [172 + (24 \times -7)] = 50$
- 11** **a** $a + 4d = 23$ (1)
 $\frac{10}{2} (2a + 9d) = 240 \Rightarrow 2a + 9d = 48$
 $2 \times (1) \Rightarrow 2a + 8d = 46$
subtracting, $d = 2$
sub. $a = 15$
b $S_{60} = \frac{60}{2} [30 + (59 \times 2)] = 4440$
- 12** **a** $S_n = 1 + 2 + 3 + \dots + (n - 1) + n$
write in reverse
 $S_n = n + (n - 1) + \dots + 3 + 2 + 1$
adding, $2S_n = n \times (n + 1)$
 $S_n = \frac{1}{2} n(n + 1)$
b $= S_{100} - S_{29}$
 $= \frac{1}{2} \times 100 \times 101 - \frac{1}{2} \times 29 \times 30$
 $= 5050 - 435 = 4615$

- 13** a $5 + 7 + 9 + 11 + 13$
 b $15 + 12 + 9 + 6 + 3 + 0 - 3 - 6 - 9$
 c $15 + 19 + 23 + 27 + 31 + 35 + 39$
 d $4\frac{1}{2} + 4 + 3\frac{1}{2} + 3 + 2\frac{1}{2} + 2 + 1\frac{1}{2} + 1$
- 14** a AP: $a = 4$,
 $l = 61, n = 20$
 $S_{20} = \frac{20}{2}(4 + 61)$
 $= 650$
 b AP: $a = 88$,
 $l = 0, n = 45$
 $S_{45} = \frac{45}{2}(88 + 0)$
 $= 1980$
 c AP: $a = 19$,
 $l = 127, n = 28$
 $S_{28} = \frac{28}{2}(19 + 127)$
 $= 2044$
 d AP: $a = 3$,
 $l = 13, n = 41$
 $S_{41} = \frac{41}{2}(3 + 13)$
 $= 328$
- 15** AP: $a = -2, l = 4n - 6$
 $S_n = \frac{n}{2}[-2 + (4n - 6)] = 720$
 $\therefore n(4n - 8) = 1440$
 $n^2 - 2n - 360 = 0$
 $(n + 18)(n - 20) = 0$
 $n > 0 \therefore n = 20$
- 16** a AP: $a = 2, l = 160, n = 80$
 $S_{80} = \frac{80}{2}(2 + 160) = 6480$
 b AP: $a = 3, l = 198, n = 66$
 $S_{66} = \frac{66}{2}(3 + 198) = 6633$
 c AP: $a = 30, l = 300, d = 6$
 $30 + 6(n - 1) = 300 \therefore n = 46$
 $S_{46} = \frac{46}{2}(30 + 300) = 7590$
- 17** a $a + (9 \times -11) = 101$
 $a = 200$
 b $S_{30} = \frac{30}{2}[400 + (29 \times -11)] = 1215$
- 18** a $a = 17, 17 + 4d = 27 \therefore d = 2.5$
 b $17 + 2.5(r - 1) = 132$
 $r = 47$
 c $S_{47} = \frac{47}{2}(17 + 132) = 3501.5$
- 19** a $\frac{6}{2}(2a + 5d) = 213 \Rightarrow 2a + 5d = 71$
 $\frac{10}{2}(2a + 9d) = 295 \Rightarrow 2a + 9d = 59$
 subtracting, $4d = -12$
 $d = -3$
 sub. $a = 43$
 b $43 - 3(n - 1) > 0$
 $n < \frac{46}{3} \therefore 15$ positive terms
 c max S_n when $n = 15$
 $S_{15} = \frac{15}{2}[86 + (14 \times -3)] = 330$
- 20** a $S_8 = (2 \times 8^2) + (5 \times 8) = 168$
 b $S_7 = (2 \times 7^2) + (5 \times 7) = 133$
 $u_8 = S_8 - S_7 = 35$
 c $S_{n-1} = 2(n-1)^2 + 5(n-1)$
 $= 2n^2 + n - 3$
 $u_n = S_n - S_{n-1}$
 $= (2n^2 + 5n) - (2n^2 + n - 3)$
 $= 4n + 3$
- 21** a $(2k + 3) - (k + 2) = (4k - 2) - (2k + 3)$
 $k + 1 = 2k - 5$
 $k = 6$
 b $a = 8, a + d = 15 \therefore d = 7$
 $S_{25} = \frac{25}{2}[16 + (24 \times 7)] = 2300$
- 22** a $2t - (5 - t) = (6t - 3) - 2t$
 $3t - 5 = 4t - 3$
 $t = -2$
 b $u_5 = 7, u_6 = -4 \therefore d = -11$
 $a + (4 \times -11) = 7 \therefore a = 51$
 $S_{18} = \frac{18}{2}[102 + (17 \times -11)] = -765$