SEQUENCES AND SERIES

1 Write down the first five terms of the sequences with nth terms, u_n, given for $n \geq 1$ by

a $u_n = 4n + 5$ **b** $u_n = (n+1)^2$ **c** $u_n = 2^n$ **d** $u_n = \frac{n}{n+1}$

e $u_n = n^3 - 2n$ **f** $u_n = 1 - \frac{1}{3}n$ **g** $u_n = 1 - \frac{1}{2n}$ **h** $u_n = 32 \times (\frac{1}{2})^n$

2 The nth term of each of the following sequences is given by $u_n = an + b$, for $n \geq 1$.
Find the values of the constants a and b in each case.

a 4, 7, 10, 13, 16, … **b** 0, 7, 14, 21, 28, … **c** 16, 14, 12, 10, 8, …

d 0.4, 1.7, 3.0, 4.3, 5.6, … **e** 100, 83, 66, 49, 32, … **f** –13, –5, 3, 11, 19, …

3 Find a possible expression for the nth term of each of the following sequences.

a 1, 6, 11, 16, 21, … **b** 3, 9, 27, 81, 243, … **c** 2, 8, 18, 32, 50, …

d $\frac{1}{2}$, 1, 2, 4, 8, … **e** 22, 11, 0, –11, –22, … **f** 0, 1, 8, 27, 64, …

g 4, 7, 12, 19, 28, … **h** $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \ldots$ **i** 1, 3, 7, 15, 31, …

4 The nth term of a sequence, u_n, is given by

$$u_n = c + 3^{n-2}.$$

Given that $u_3 = 11$,

a find the value of the constant c,

b find the value of u_6.

5 The nth term of a sequence, u_n, is given by

$$u_n = n(2n + k).$$

Given that $u_6 = 2u_4 - 2$,

a find the value of the constant k,

b prove that for all values of n, $u_n - u_{n-1} = 4n + 3$.

6 The nth term of a sequence, u_n, is given by

$$u_n = k^n - 3.$$

Given that $u_1 + u_2 = 0$,

a find the two possible values of the constant k.

b For each value of k found in part **a**, find the corresponding value of u_5.

7 Write down the first four terms of each sequence.

a $u_n = u_{n-1} + 4$, $n > 1$, $u_1 = 3$ **b** $u_n = 3u_{n-1} + 1$, $n > 1$, $u_1 = 2$

c $u_{n+1} = 2u_n + 5$, $n > 0$, $u_1 = -2$ **d** $u_n = 7 - u_{n-1}$, $n \geq 2$, $u_1 = 5$

e $u_n = 2(5 - 2u_{n-1})$, $n > 1$, $u_1 = -1$ **f** $u_n = \frac{1}{10}(u_{n-1} + 20)$, $n \geq 2$, $u_1 = 10$

g $u_{n+1} = 1 - \frac{1}{3}u_n$, $n \geq 1$, $u_1 = 6$ **h** $u_{n+1} = \frac{1}{2 + u_n}$, $n \geq 1$, $u_1 = 0$

8 In each case, write down a recurrence relation that would produce the given sequence.

a 5, 9, 13, 17, 21, … **b** 1, 3, 9, 27, 81, … **c** 62, 44, 26, 8, −10, …

d 120, 60, 30, 15, 7.5, … **e** 4, 9, 19, 39, 79, … **f** 1, 3, 11, 43, 171, …

9 Given that the following sequences can be defined by recurrence relations of the form $u_n = au_{n-1} + b$, $n > 1$, find the values of the constants a and b for each sequence.

a –4, –3, –1, 3, 11, … **b** 0, 8, 4, 6, 5, … **c** $7\frac{3}{4}$, $5\frac{1}{2}$, 4, 3, $2\frac{1}{3}$, …

10 For each of the following sequences, find expressions for u_2 and u_3 in terms of the constant k.

a $u_n = 4u_{n-1} + 3k$, $n > 1$, $u_1 = 1$ **b** $u_{n+1} = ku_n + 5$, $n > 0$, $u_1 = 2$

c $u_n = 4u_{n-1} - k$, $n > 1$, $u_1 = k$ **d** $u_n = 2 - ku_{n-1}$, $n \geq 2$, $u_1 = -1$

e $u_{n+1} = \frac{u_n}{k}$, $n \geq 1$, $u_1 = 4$ **f** $u_{n+1} = \sqrt[3]{61k^3 + u_n{}^3}$, $n > 0$, $u_1 = k\sqrt[3]{3}$

11 A sequence is given by the recurrence relation

$$u_n = \tfrac{1}{2}(k + 3u_{n-1}), \quad n > 1, \quad u_1 = 2.$$

a Find an expression for u_3 in terms of the constant k.

Given that $u_3 = 7$,

b find the value of k and the value of u_4.

12 For the sequences given by the following recurrence relations, find u_4 and u_1.

a $u_n = 3u_{n-1} - 2$, $n > 1$, $u_3 = 10$ **b** $u_{n+1} = \frac{3}{4}u_n + 2$, $n > 0$, $u_3 = 5$

c $u_{n+1} = 0.2(1 - u_n)$, $n > 0$, $u_3 = -0.2$ **d** $u_n = \frac{1}{2}\sqrt{u_{n-1}}$, $n > 1$, $u_3 = 1$

13 A sequence is defined by

$$u_{n+1} = u_n + c, \quad n \geq 1, \quad u_1 = 2,$$

where c is a constant. Given that $u_5 = 30$, find

a the value of c,

b an expression for u_n in terms of n.

14 The terms of a sequence u_1, u_2, u_3, … are given by

$$u_n = 3(u_{n-1} - k), \quad n > 1,$$

where k is a constant. Given that $u_1 = -4$,

a find expressions for u_2 and u_3 in terms of k.

Given also that $u_3 = 7u_2 + 3$, find

b the value of k,

c the value of u_4.

15 A sequence of terms $\{t_n\}$ is defined, for $n > 1$, by the recurrence relation

$$t_n = kt_{n-1} + 2,$$

where k is a constant. Given that $t_1 = 1.5$,

a find expressions for t_2 and t_3 in terms of k.

Given also that $t_3 = 12$,

b find the possible values of k.