


**SERIES**
**Answers**

- 1**
- a**  $= 1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \frac{(-1)(-2)(-3)}{3 \times 2}x^3 + \dots$   
 $= 1 - x + x^2 - x^3 + \dots$
- b**  $= 1 + (\frac{1}{2})x + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}x^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3 \times 2}x^3 + \dots$   
 $= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots$
- c**  $= 2[1 + (-3)x + \frac{(-3)(-4)}{2}x^2 + \frac{(-3)(-4)(-5)}{3 \times 2}x^3 + \dots]$   
 $= 2 - 6x + 12x^2 - 20x^3 + \dots$
- d**  $= 1 + (\frac{2}{3})x + \frac{(\frac{2}{3})(-\frac{1}{3})}{2}x^2 + \frac{(\frac{2}{3})(-\frac{1}{3})(-\frac{4}{3})}{3 \times 2}x^3 + \dots$   
 $= 1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 + \dots$
- e**  $= (1 - x)^{\frac{1}{3}} = 1 + (\frac{1}{3})(-x) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2}(-x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3 \times 2}(-x)^3 + \dots$   
 $= 1 - \frac{1}{3}x - \frac{1}{9}x^2 - \frac{5}{81}x^3 + \dots$
- f**  $= (1 + x)^{-2} = 1 + (-2)x + \frac{(-2)(-3)}{2}x^2 + \frac{(-2)(-3)(-4)}{3 \times 2}x^3 + \dots$   
 $= 1 - 2x + 3x^2 - 4x^3 + \dots$
- g**  $= \frac{1}{4}(1 - x)^{-4} = \frac{1}{4}[1 + (-4)(-x) + \frac{(-4)(-5)}{2}(-x)^2 + \frac{(-4)(-5)(-6)}{3 \times 2}(-x)^3 + \dots]$   
 $= \frac{1}{4} + x + \frac{5}{2}x^2 + 5x^3 + \dots$
- h**  $= 3(1 - x)^{-\frac{1}{2}} = 3[1 + (-\frac{1}{2})(-x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3 \times 2}(-x)^3 + \dots]$   
 $= 3 + \frac{3}{2}x + \frac{9}{8}x^2 + \frac{15}{16}x^3 + \dots$
- 2**
- a**  $= 1 + (\frac{1}{2})(2x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(2x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3 \times 2}(2x)^3 + \dots$   
 $= 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots, |2x| < 1 \quad \therefore \text{valid for } |x| < \frac{1}{2}$
- b**  $= 1 + (-1)(-3x) + \frac{(-1)(-2)}{2}(-3x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-3x)^3 + \dots$   
 $= 1 + 3x + 9x^2 + 27x^3 + \dots, |-3x| < 1 \quad \therefore \text{valid for } |x| < \frac{1}{3}$
- c**  $= 1 + (-\frac{1}{2})(-4x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-4x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3 \times 2}(-4x)^3 + \dots$   
 $= 1 + 2x + 6x^2 + 20x^3 + \dots, |-4x| < 1 \quad \therefore \text{valid for } |x| < \frac{1}{4}$
- d**  $= 1 + (-3)(\frac{1}{2}x) + \frac{(-3)(-4)}{2}(\frac{1}{2}x)^2 + \frac{(-3)(-4)(-5)}{3 \times 2}(\frac{1}{2}x)^3 + \dots$   
 $= 1 - \frac{3}{2}x + \frac{3}{2}x^2 - \frac{5}{4}x^3 + \dots, |\frac{1}{2}x| < 1 \quad \therefore \text{valid for } |x| < 2$
- e**  $= 1 + (\frac{1}{3})(-6x) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2}(-6x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3 \times 2}(-6x)^3 + \dots$   
 $= 1 - 2x - 4x^2 - \frac{40}{3}x^3 + \dots, |-6x| < 1 \quad \therefore \text{valid for } |x| < \frac{1}{6}$
- f**  $= 1 + (-4)(\frac{1}{4}x) + \frac{(-4)(-5)}{2}(\frac{1}{4}x)^2 + \frac{(-4)(-5)(-6)}{3 \times 2}(\frac{1}{4}x)^3 + \dots$   
 $= 1 - x + \frac{5}{8}x^2 - \frac{5}{16}x^3 + \dots, |\frac{1}{4}x| < 1 \quad \therefore \text{valid for } |x| < 4$
- g**  $= 1 + (\frac{3}{2})(2x) + \frac{(\frac{3}{2})(\frac{1}{2})}{2}(2x)^2 + \frac{(\frac{3}{2})(\frac{1}{2})(-\frac{1}{2})}{3 \times 2}(2x)^3 + \dots$   
 $= 1 + 3x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \dots, |2x| < 1 \quad \therefore \text{valid for } |x| < \frac{1}{2}$
- h**  $= 1 + (-\frac{4}{3})(-3x) + \frac{(-\frac{4}{3})(-\frac{7}{3})}{2}(-3x)^2 + \frac{(-\frac{4}{3})(-\frac{7}{3})(-\frac{10}{3})}{3 \times 2}(-3x)^3 + \dots$   
 $= 1 + 4x + 14x^2 + \frac{140}{3}x^3 + \dots, |-3x| < 1 \quad \therefore \text{valid for } |x| < \frac{1}{3}$

SERIES	Answers	page 2
<b>3</b>	$\mathbf{a} = 1 + \left(\frac{1}{2}\right)(-2x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(-2x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3 \times 2}(-2x)^3 + \dots$ $= 1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3 + \dots$	
	$\mathbf{b} \quad \sqrt{0.98} = (1 - 2x)^{\frac{1}{2}} \text{ when } x = 0.01$ $\therefore \sqrt{0.98} \approx 1 - (0.01) - \frac{1}{2}(0.01)^2 - \frac{1}{2}(0.01)^3$ $= 1 - 0.01 - 0.000\,05 - 0.000\,000\,5$ $= 0.989\,949\,5$	
	$\mathbf{c} \quad \sqrt{0.98} = \sqrt{\frac{98}{100}} = \sqrt{\frac{49 \times 2}{100}} = \frac{7}{10} \sqrt{2}$ $\therefore \sqrt{2} \approx \frac{10}{7} \times 0.989\,949\,5 = 1.414\,213\,6 \text{ (8sf)}$	
<b>4</b>	$\mathbf{a} = 2^{-1}\left(1 + \frac{1}{2}x\right)^{-1} = \frac{1}{2}\left(1 + \frac{1}{2}x\right)^{-1}$ $= \frac{1}{2}\left[1 + (-1)\left(\frac{1}{2}x\right) + \frac{(-1)(-2)}{2}\left(\frac{1}{2}x\right)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}\left(\frac{1}{2}x\right)^3 + \dots\right]$ $= \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \dots, \left \frac{1}{2}x\right  < 1 \quad \therefore \text{valid for }  x  < 2$	
	$\mathbf{b} = 4^{\frac{1}{2}}\left(1 + \frac{1}{4}x\right)^{\frac{1}{2}} = 2\left(1 + \frac{1}{4}x\right)^{\frac{1}{2}}$ $= 2\left[1 + \left(\frac{1}{2}\right)\left(\frac{1}{4}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}\left(\frac{1}{4}x\right)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3 \times 2}\left(\frac{1}{4}x\right)^3 + \dots\right]$ $= 2 + \frac{1}{4}x - \frac{1}{64}x^2 + \frac{1}{512}x^3 + \dots, \left \frac{1}{4}x\right  < 1 \quad \therefore \text{valid for }  x  < 4$	
	$\mathbf{c} = 3^{-3}\left(1 - \frac{1}{3}x\right)^{-3} = \frac{1}{27}\left(1 - \frac{1}{3}x\right)^{-3}$ $= \frac{1}{27}\left[1 + (-3)\left(-\frac{1}{3}x\right) + \frac{(-3)(-4)}{2}\left(-\frac{1}{3}x\right)^2 + \frac{(-3)(-4)(-5)}{3 \times 2}\left(-\frac{1}{3}x\right)^3 + \dots\right]$ $= \frac{1}{27} + \frac{1}{27}x + \frac{2}{81}x^2 + \frac{10}{729}x^3 + \dots, \left -\frac{1}{3}x\right  < 1 \quad \therefore \text{valid for }  x  < 3$	
	$\mathbf{d} = 9^{\frac{1}{2}}\left(1 + \frac{1}{3}x\right)^{\frac{1}{2}} = 3\left(1 + \frac{1}{3}x\right)^{\frac{1}{2}}$ $= 3\left[1 + \left(\frac{1}{2}\right)\left(\frac{1}{3}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}\left(\frac{1}{3}x\right)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3 \times 2}\left(\frac{1}{3}x\right)^3 + \dots\right]$ $= 3 + \frac{1}{2}x - \frac{1}{24}x^2 + \frac{1}{144}x^3 + \dots, \left \frac{1}{3}x\right  < 1 \quad \therefore \text{valid for }  x  < 3$	
	$\mathbf{e} = 8^{\frac{1}{3}}\left(1 - 3x\right)^{\frac{1}{3}} = 2\left(1 - 3x\right)^{\frac{1}{3}}$ $= 2\left[1 + \left(\frac{1}{3}\right)(-3x) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2}(-3x)^2 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3 \times 2}(-3x)^3 + \dots\right]$ $= 2 - 2x - 2x^2 - \frac{10}{3}x^3 + \dots,  -3x  < 1 \quad \therefore \text{valid for }  x  < \frac{1}{3}$	
	$\mathbf{f} = 4^{-1}\left(1 - \frac{3}{4}x\right)^{-1} = \frac{1}{4}\left(1 - \frac{3}{4}x\right)^{-1}$ $= \frac{1}{4}\left[1 + (-1)\left(-\frac{3}{4}x\right) + \frac{(-1)(-2)}{2}\left(-\frac{3}{4}x\right)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}\left(-\frac{3}{4}x\right)^3 + \dots\right]$ $= \frac{1}{4} + \frac{3}{16}x + \frac{9}{64}x^2 + \frac{27}{256}x^3 + \dots, \left -\frac{3}{4}x\right  < 1 \quad \therefore \text{valid for }  x  < \frac{4}{3}$	
	$\mathbf{g} = 4^{-\frac{1}{2}}\left(1 + \frac{3}{2}x\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1 + \frac{3}{2}x\right)^{-\frac{1}{2}}$ $= \frac{1}{2}\left[1 + \left(-\frac{1}{2}\right)\left(\frac{3}{2}x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(\frac{3}{2}x\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3 \times 2}\left(\frac{3}{2}x\right)^3 + \dots\right]$ $= \frac{1}{2} - \frac{3}{8}x + \frac{27}{64}x^2 - \frac{135}{256}x^3 + \dots, \left \frac{3}{2}x\right  < 1 \quad \therefore \text{valid for }  x  < \frac{2}{3}$	
	$\mathbf{h} = 3^{-2}\left(1 + \frac{2}{3}x\right)^{-2} = \frac{1}{9}\left(1 + \frac{2}{3}x\right)^{-2}$ $= \frac{1}{9}\left[1 + (-2)\left(\frac{2}{3}x\right) + \frac{(-2)(-3)}{2}\left(\frac{2}{3}x\right)^2 + \frac{(-2)(-3)(-4)}{3 \times 2}\left(\frac{2}{3}x\right)^3 + \dots\right]$ $= \frac{1}{9} - \frac{4}{27}x + \frac{4}{27}x^2 - \frac{32}{243}x^3 + \dots, \left \frac{2}{3}x\right  < 1 \quad \therefore \text{valid for }  x  < \frac{3}{2}$	

SERIES	Answers	page 3
--------	---------	--------

- 5**
- a**  $= 1 + (-1)(2x) + \frac{(-1)(-2)}{2} (2x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2} (2x)^3 + \dots$   
 $= 1 - 2x + 4x^2 - 8x^3 + \dots$
- b**  $= (1-x)(1+2x)^{-1} = (1-x)(1 - 2x + 4x^2 - 8x^3 + \dots)$   
 $= 1 - 2x + 4x^2 - 8x^3 - x + 2x^2 - 4x^3 + \dots$   
 $= 1 - 3x + 6x^2 - 12x^3 + \dots$
- 6**
- a**  $= (1+3x)(1-x)^{-1} = (1+3x)[1 + (-1)(-x) + \frac{(-1)(-2)}{2} (-x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2} (-x)^3 + \dots]$   
 $= (1+3x)(1+x+x^2+x^3+\dots)$   
 $= 1+x+x^2+x^3+3x+3x^2+3x^3+\dots$   
 $= 1+4x+4x^2+4x^3+\dots, | -x | < 1 \quad \therefore \text{valid for } |x| < 1$
- b**  $= (2x-1)(1+4x)^{-2} = (2x-1)[1 + (-2)(4x) + \frac{(-2)(-3)}{2} (4x)^2 + \frac{(-2)(-3)(-4)}{3 \times 2} (4x)^3 + \dots]$   
 $= (2x-1)(1-8x+48x^2-256x^3+\dots)$   
 $= 2x-16x^2+96x^3-1+8x-48x^2+256x^3+\dots$   
 $= -1+10x-64x^2+352x^3+\dots, |4x| < 1 \quad \therefore \text{valid for } |x| < \frac{1}{4}$
- c**  $= (3+x)(2-x)^{-1} = (3+x) \times 2^{-1} (1 - \frac{1}{2}x)^{-1}$   
 $= (3+x) \times \frac{1}{2} [1 + (-1)(-\frac{1}{2}x) + \frac{(-1)(-2)}{2} (-\frac{1}{2}x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2} (-\frac{1}{2}x)^3 + \dots]$   
 $= (3+x)(\frac{1}{2} + \frac{1}{4}x + \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots)$   
 $= \frac{3}{2} + \frac{3}{4}x + \frac{3}{8}x^2 + \frac{3}{16}x^3 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \dots$   
 $= \frac{3}{2} + \frac{5}{4}x + \frac{5}{8}x^2 + \frac{5}{16}x^3 + \dots, |-\frac{1}{2}x| < 1 \quad \therefore \text{valid for } |x| < 2$
- d**  $= (1-x)(1+2x)^{-\frac{1}{2}} = (1-x)[1 + (-\frac{1}{2})(2x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2} (2x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3 \times 2} (2x)^3 + \dots]$   
 $= (1-x)(1-x+\frac{3}{2}x^2-\frac{5}{2}x^3+\dots)$   
 $= 1-x+\frac{3}{2}x^2-\frac{5}{2}x^3-x+x^2-\frac{3}{2}x^3+\dots$   
 $= 1-2x+\frac{5}{2}x^2-4x^3+\dots, |2x| < 1 \quad \therefore \text{valid for } |x| < \frac{1}{2}$
- 7**
- a**  $\frac{x-2}{(1-x)(1-2x)} \equiv \frac{A}{1-x} + \frac{B}{1-2x}$   
 $x-2 \equiv A(1-2x) + B(1-x)$   
 $x=1 \quad \Rightarrow \quad -1 = -A \quad \Rightarrow \quad A=1$   
 $x=\frac{1}{2} \quad \Rightarrow \quad -\frac{3}{2} = \frac{1}{2}B \quad \Rightarrow \quad B=-3$   
 $\therefore \frac{x-2}{(1-x)(1-2x)} \equiv \frac{1}{1-x} - \frac{3}{1-2x}$
- b**  $\frac{1}{1-x} = (1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2} (-x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2} (-x)^3 + \dots$   
 $= 1+x+x^2+x^3+\dots, | -x | < 1 \quad \therefore |x| < 1$   
 $\frac{3}{1-2x} = 3(1-2x)^{-1} = 3[1 + (-1)(-2x) + \frac{(-1)(-2)}{2} (-2x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2} (-2x)^3 + \dots]$   
 $= 3+6x+12x^2+24x^3+\dots, | -2x | < 1 \quad \therefore |x| < \frac{1}{2}$   
 $\therefore \frac{x-2}{(1-x)(1-2x)} = (1+x+x^2+x^3+\dots) - (3+6x+12x^2+24x^3+\dots)$   
 $= -2-5x-11x^2-23x^3+\dots, \text{ valid for } |x| < \frac{1}{2}$

**8 a**  $\frac{4}{(1+x)(1-3x)} \equiv \frac{A}{1+x} + \frac{B}{1-3x}$   
 $4 \equiv A(1-3x) + B(1+x)$   
 $x = -1 \Rightarrow 4 = 4A \Rightarrow A = 1$   
 $x = \frac{1}{3} \Rightarrow 4 = \frac{4}{3}B \Rightarrow B = 3$   
 $\therefore f(x) \equiv \frac{1}{1+x} + \frac{3}{1-3x}$   
 $\frac{1}{1+x} = (1+x)^{-1} = 1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \frac{(-1)(-2)(-3)}{3 \times 2}x^3 + \dots$   
 $= 1 - x + x^2 - x^3 + \dots, |x| < 1$   
 $\frac{3}{1-3x} = 3(1-3x)^{-1} = 3[1 + (-1)(-3x) + \frac{(-1)(-2)}{2}(-3x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-3x)^3 + \dots]$   
 $= 3 + 9x + 27x^2 + 81x^3 + \dots, |-3x| < 1 \therefore |x| < \frac{1}{3}$   
 $\therefore f(x) \equiv (1 - x + x^2 - x^3 + \dots) + (3 + 9x + 27x^2 + 81x^3 + \dots)$   
 $f(x) \equiv 4 + 8x + 28x^2 + 80x^3 + \dots, \text{ valid for } |x| < \frac{1}{3}$

**b**  $\frac{1-6x}{1+3x-4x^2} \equiv \frac{1-6x}{(1-x)(1+4x)} \equiv \frac{A}{1-x} + \frac{B}{1+4x}$   
 $1-6x \equiv A(1+4x) + B(1-x)$   
 $x = 1 \Rightarrow -5 = 5A \Rightarrow A = -1$   
 $x = -\frac{1}{4} \Rightarrow \frac{5}{2} = \frac{5}{4}B \Rightarrow B = 2$   
 $\therefore f(x) \equiv \frac{2}{1+4x} - \frac{1}{1-x}$   
 $\frac{2}{1+4x} = 2(1+4x)^{-1} = 2[1 + (-1)(4x) + \frac{(-1)(-2)}{2}(4x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(4x)^3 + \dots]$   
 $= 2 - 8x + 32x^2 - 128x^3 + \dots, |4x| < 1 \therefore |x| < \frac{1}{4}$   
 $\frac{1}{1-x} = (1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-x)^3 + \dots$   
 $= 1 + x + x^2 + x^3 + \dots, |-x| < 1 \therefore |x| < 1$   
 $\therefore f(x) \equiv (2 - 8x + 32x^2 - 128x^3 + \dots) - (1 + x + x^2 + x^3 + \dots)$   
 $f(x) \equiv 1 - 9x + 31x^2 - 129x^3 + \dots, \text{ valid for } |x| < \frac{1}{4}$

**c**  $\frac{5}{2-3x-2x^2} \equiv \frac{5}{(1-2x)(2+x)} = \frac{A}{1-2x} + \frac{B}{2+x}$   
 $5 \equiv A(2+x) + B(1-2x)$   
 $x = \frac{1}{2} \Rightarrow 5 = \frac{5}{2}A \Rightarrow A = 2$   
 $x = -2 \Rightarrow 5 = 5B \Rightarrow B = 1$   
 $\therefore f(x) \equiv \frac{2}{1-2x} + \frac{1}{2+x}$   
 $\frac{2}{1-2x} = 2(1-2x)^{-1} = 2[1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-2x)^3 + \dots]$   
 $= 2 + 4x + 8x^2 + 16x^3 + \dots, |-2x| < 1 \therefore |x| < \frac{1}{2}$   
 $\frac{1}{2+x} = (2+x)^{-1} = 2^{-1}(1 + \frac{1}{2}x)^{-1} = \frac{1}{2}[1 + (-1)(\frac{1}{2}x) + \frac{(-1)(-2)}{2}(\frac{1}{2}x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(\frac{1}{2}x)^3 + \dots]$   
 $= \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \dots, |\frac{1}{2}x| < 1 \therefore |x| < 2$   
 $\therefore f(x) \equiv (2 + 4x + 8x^2 + 16x^3 + \dots) + (\frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \dots)$   
 $f(x) \equiv \frac{5}{2} + \frac{15}{4}x + \frac{65}{8}x^2 + \frac{255}{16}x^3 + \dots, \text{ valid for } |x| < \frac{1}{2}$

$$\mathbf{d} \quad \frac{7x-3}{x^2-4x+3} \equiv \frac{7x-3}{(x-1)(x-3)} \equiv \frac{A}{x-1} + \frac{B}{x-3}$$

$$7x-3 \equiv A(x-3) + B(x-1)$$

$$x=1 \quad \Rightarrow \quad 4 = -2A \quad \Rightarrow \quad A = -2$$

$$x=3 \quad \Rightarrow \quad 18 = 2B \quad \Rightarrow \quad B = 9$$

$$\therefore f(x) \equiv \frac{9}{x-3} - \frac{2}{x-1} \equiv \frac{2}{1-x} - \frac{9}{3-x}$$

$$\frac{2}{1-x} = 2(1-x)^{-1} = 2[1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-x)^3 + \dots]$$

$$= 2 + 2x + 2x^2 + 2x^3 + \dots, \quad |-x| < 1 \quad \therefore |x| < 1$$

$$\frac{9}{3-x} = 9(3-x)^{-1} = 9 \times 3^{-1} (1 - \frac{1}{3}x)^{-1}$$

$$= 3[1 + (-1)(-\frac{1}{3}x) + \frac{(-1)(-2)}{2}(-\frac{1}{3}x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-\frac{1}{3}x)^3 + \dots]$$

$$= 3 + x + \frac{1}{3}x^2 + \frac{1}{9}x^3 + \dots, \quad |-\frac{1}{3}x| < 1 \quad \therefore |x| < 3$$

$$\therefore f(x) \equiv (2 + 2x + 2x^2 + 2x^3 + \dots) - (3 + x + \frac{1}{3}x^2 + \frac{1}{9}x^3 + \dots)$$

$$f(x) \equiv -1 + x + \frac{5}{3}x^2 + \frac{17}{9}x^3 + \dots, \quad \text{valid for } |x| < 1$$

$$\mathbf{e} \quad \frac{3+5x}{(1+3x)(1+x)^2} \equiv \frac{A}{1+3x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$$

$$3 + 5x \equiv A(1+x)^2 + B(1+3x)(1+x) + C(1+3x)$$

$$x = -\frac{1}{3} \quad \Rightarrow \quad \frac{4}{3} = \frac{4}{9}A \quad \Rightarrow \quad A = 3$$

$$x = -1 \quad \Rightarrow \quad -2 = -2C \quad \Rightarrow \quad C = 1$$

$$\text{coeffs of } x^2 \Rightarrow 0 = A + 3B \quad \Rightarrow \quad B = -1$$

$$\therefore f(x) \equiv \frac{3}{1+3x} - \frac{1}{1+x} + \frac{1}{(1+x)^2}$$

$$\frac{3}{1+3x} = 3(1+3x)^{-1} = 3[1 + (-1)(3x) + \frac{(-1)(-2)}{2}(3x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(3x)^3 + \dots]$$

$$= 3 - 9x + 27x^2 - 81x^3 + \dots, \quad |3x| < 1 \quad \therefore |x| < \frac{1}{3}$$

$$\frac{1}{1+x} = (1+x)^{-1} = 1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \frac{(-1)(-2)(-3)}{3 \times 2}x^3 + \dots$$

$$= 1 - x + x^2 - x^3 + \dots, \quad |x| < 1$$

$$\frac{1}{(1+x)^2} = (1+x)^{-2} = 1 + (-2)x + \frac{(-2)(-3)}{2}x^2 + \frac{(-2)(-3)(-4)}{3 \times 2}x^3 + \dots$$

$$= 1 - 2x + 3x^2 - 4x^3 + \dots, \quad |x| < 1$$

$$\therefore f(x) \equiv (3 - 9x + 27x^2 - 81x^3 + \dots) - (1 - x + x^2 - x^3 + \dots) + (1 - 2x + 3x^2 - 4x^3 + \dots)$$

$$f(x) \equiv 3 - 10x + 29x^2 - 84x^3 + \dots, \quad \text{valid for } |x| < \frac{1}{3}$$

f

$$2x^2 + x - 1 \overline{) \begin{array}{r} 2x^2 + 0x + 4 \\ 2x^2 + x - 1 \\ \hline -x + 5 \end{array}}$$

$$\therefore \frac{2x^2 + 4}{2x^2 + x - 1} \equiv 1 + \frac{5 - x}{2x^2 + x - 1}$$

$$\frac{5 - x}{2x^2 + x - 1} \equiv \frac{5 - x}{(2x - 1)(x + 1)} \equiv \frac{A}{2x - 1} + \frac{B}{x + 1}$$

$$5 - x \equiv A(x + 1) + B(2x - 1)$$

$$x = \frac{1}{2} \Rightarrow \frac{9}{2} = \frac{3}{2}A \Rightarrow A = 3$$

$$x = -1 \Rightarrow 6 = -3B \Rightarrow B = -2$$

$$\therefore f(x) \equiv 1 + \frac{3}{2x - 1} - \frac{2}{x + 1} \equiv 1 - \frac{3}{1 - 2x} - \frac{2}{1 + x}$$

$$\frac{3}{1 - 2x} = 3(1 - 2x)^{-1} = 3[1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-2x)^3 + \dots]$$

$$= 3 + 6x + 12x^2 + 24x^3 + \dots, \quad |-2x| < 1 \quad \therefore |x| < \frac{1}{2}$$

$$\frac{2}{1 + x} = 2(1 + x)^{-1} = 2[1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \frac{(-1)(-2)(-3)}{3 \times 2}x^3 + \dots]$$

$$= 2 - 2x + 2x^2 - 2x^3 + \dots, \quad |x| < 1$$

$$\therefore f(x) \equiv 1 - (3 + 6x + 12x^2 + 24x^3 + \dots) - (2 - 2x + 2x^2 - 2x^3 + \dots)$$

$$f(x) \equiv -4 - 4x - 14x^2 - 22x^3 + \dots, \quad \text{valid for } |x| < \frac{1}{2}$$