# SERIES

**1** Find the binomial expansion of each of the following in ascending powers of $x$ up to and including the term in $x^3$, for $|x| < 1$.

**a** $(1 + x)^{-1}$ **b** $(1 + x)^{\frac{1}{2}}$ **c** $2(1 + x)^{-3}$ **d** $(1 + x)^{\frac{2}{3}}$

**e** $\sqrt[3]{1-x}$ **f** $\dfrac{1}{(1+x)^2}$ **g** $\dfrac{1}{4(1-x)^4}$ **h** $\dfrac{3}{\sqrt{1-x}}$

**2** Expand each of the following in ascending powers of $x$ up to and including the term in $x^3$ and state the set of values of $x$ for which each expansion is valid.

**a** $(1 + 2x)^{\frac{1}{2}}$ **b** $(1 - 3x)^{-1}$ **c** $(1 - 4x)^{-\frac{1}{2}}$ **d** $(1 + \frac{1}{2}x)^{-3}$

**e** $(1 - 6x)^{\frac{1}{3}}$ **f** $(1 + \frac{1}{4}x)^{-4}$ **g** $(1 + 2x)^{\frac{3}{2}}$ **h** $(1 - 3x)^{-\frac{4}{3}}$

**3** **a** Expand $(1 - 2x)^{\frac{1}{2}}$, $|x| < \frac{1}{2}$, in ascending powers of $x$ up to and including the term in $x^3$.

**b** By substituting a suitable value of $x$ in your expansion, find an estimate for $\sqrt{0.98}$

**c** Show that $\sqrt{0.98} = \frac{7}{10}\sqrt{2}$ and hence find the value of $\sqrt{2}$ correct to 8 significant figures.

**4** Expand each of the following in ascending powers of $x$ up to and including the term in $x^3$ and state the set of values of $x$ for which each expansion is valid.

**a** $(2 + x)^{-1}$ **b** $(4 + x)^{\frac{1}{2}}$ **c** $(3 - x)^{-3}$ **d** $(9 + 3x)^{\frac{1}{2}}$

**e** $(8 - 24x)^{\frac{1}{3}}$ **f** $(4 - 3x)^{-1}$ **g** $(4 + 6x)^{-\frac{1}{2}}$ **h** $(3 + 2x)^{-2}$

**5** **a** Expand $(1 + 2x)^{-1}$, $|x| < \frac{1}{2}$, in ascending powers of $x$ up to and including the term in $x^3$.

**b** Hence find the series expansion of $\dfrac{1-x}{1+2x}$, $|x| < \frac{1}{2}$, in ascending powers of $x$ up to and including the term in $x^3$.

**6** Find the first four terms in the series expansion in ascending powers of $x$ of each of the following and state the set of values of $x$ for which each expansion is valid.

**a** $\dfrac{1+3x}{1-x}$ **b** $\dfrac{2x-1}{(1+4x)^2}$ **c** $\dfrac{3+x}{2-x}$ **d** $\dfrac{1-x}{\sqrt{1+2x}}$

**7** **a** Express $\dfrac{x-2}{(1-x)(1-2x)}$ in partial fractions.

**b** Hence find the series expansion of $\dfrac{x-2}{(1-x)(1-2x)}$ in ascending powers of $x$ up to and including the term in $x^3$ and state the set of values of $x$ for which the expansion is valid.

**8** By first expressing f($x$) in partial fractions, find the series expansion of f($x$) in ascending powers of $x$ up to and including the term in $x^3$ and state the set of values of $x$ for which it is valid.

**a** $f(x) \equiv \dfrac{4}{(1+x)(1-3x)}$ **b** $f(x) \equiv \dfrac{1-6x}{1+3x-4x^2}$ **c** $f(x) \equiv \dfrac{5}{2-3x-2x^2}$

**d** $f(x) \equiv \dfrac{7x-3}{x^2-4x+3}$ **e** $f(x) \equiv \dfrac{3+5x}{(1+3x)(1+x)^2}$ **f** $f(x) \equiv \dfrac{2x^2+4}{2x^2+x-1}$