1. Find the first three terms in the expansion of $(9-16x)^{\frac{3}{2}}$ in ascending powers of *x*, and state the set of values for which this expansion is valid.

[5]

2.

i. Show that $\frac{x}{(1-x)^3} \approx x + 3x^2 + 6x^3$ for small values of *x*.

[2]

ii. Use this result, together with a suitable value of *x*, to obtain a decimal estimate of the value of $\frac{100}{729}$.

[4]

iii. Show that $\frac{x}{(1-x)^3} = -\frac{1}{x^2}\left(1-\frac{1}{x}\right)^{-3}$. Hence find the first three terms of the binomial expansion of

$\frac{x}{(1-x)^3}$ in powers of $\frac{1}{x}$.

iv. Comment on the suitability of substituting the same value of *x* as used in part (ii) in the expansion in part (iii) to estimate the value of $\frac{100}{729}$.

[1]

3.

i. Find the first three terms in the expansion of $(1-2x)^{-\frac{1}{2}}$ in ascending powers of *x*, where $|x| < \frac{1}{2}$.

[3]

ii. Hence find the coefficient of x^2 in the expansion of $\frac{x+3}{\sqrt{1-2x}}$.

[2]

4.

i. Find the first three terms in the binomial expansion of $(8-9x)^{\frac{2}{3}}$ in ascending powers of *x*.

[4]

ii. State the set of values of x for which this expansion is valid.

[1]

5. (a) Find the first three terms in the expansion of $(1+px)^{\frac{1}{3}}$ in ascending powers of x. [3]

(b) Given that the expansion of $(1+qx)(1+px)^{\frac{1}{3}}$ is

$$1+x-\tfrac{2}{9}x^2+\dots$$

find the possible values of p and q. [5]

6. (a) Find the first three terms in the expansion of $(1+2x)^{\frac{1}{2}}$ in ascending powers of x. [3]

(b) Obtain an estimate of $\sqrt{3}$ by substituting $x = 0.04$ into your answer to part (a). [3]

(c) Explain why using $x = 1$ in the expansion would not give a valid estimate of $\sqrt{3}$. [1]

7. (i) Find the first three terms in ascending powers of x in the binomial expansion of $\sqrt[4]{1+8x}$. [3]

(ii) State the range of values for which this expansion is valid. [1]

8. (a) Find the first three terms in the expansion of $(4-x)^{-\frac{1}{2}}$ in ascending powers of x. [4]

(b) The expansion of $\frac{a+bx}{\sqrt{4-x}}$ is 16 – x … . Find the values of the constants a and b. [3]

9. (a) Find the coefficient of x^4 in the expansion of $(3x-2)^{10}$. [2]

(b) In the expansion of $(1 + 2x)^n$, where n is a positive integer, the coefficients of x^7 and x^8 are equal.
Find the value of n. [3]

(c) Find the coefficient of x^3 in the expansion of $\frac{1}{\sqrt{4+x}}$. [4]

10.

(a) Expand $\sqrt{1+2x}$ in ascending powers of x, up to and including the term in x^3. [4]

(b) Hence expand $\frac{\sqrt{1+2x}}{1+9x^2}$ in ascending powers of x, up to and including the term in x^3. [3]

(c) Determine the range of values of x for which the expansion in part **(b)** is valid. [2]

END OF QUESTION paper

Mark scheme

Question				Answer/Indicative content	Marks	Part marks and guidance	
1				The first 3 marks refer to the expansion…	……….	of $\left(1-\frac{16x}{9}\right)^{3/2}$ and to no other expansion	
				First 2 terms $= 1-\frac{8}{3}x$	B1	Allow any equiv fraction for the $-\frac{8}{3}$ and ISW	$\frac{3}{2}\cdot-\frac{16}{9}$ is not an equiv fraction
				3^{rd} term $= \frac{\frac{3}{2}\cdot\frac{1}{2}}{1.2}\left(-\frac{16x}{9}\right)^2$	M1	Allow clear evidence of intention, e.g. $\frac{\frac{3}{2}\cdot\frac{1}{2}}{1.2}\frac{-16x^2}{9}$	
				$=\frac{32}{27}x^2$	A1	Allow any equiv fraction for the $\frac{32}{27}$ and ISW	
				Complete expansions ≈ $27-72x+32x^2$	A1	cao No equivalents. Ignore any further terms	If expansion $(a+b)^n$ used, award B1, B1, B1 for 27, $-72x$, $32x^2$
				valid for $\frac{-9}{16}<x<\frac{9}{16}$ or $\lvert x\rvert<\frac{9}{16}$	B1	oe Beware, e.g. $x<\left\lvert\frac{9}{16}\right\rvert$ **Examiner's Comments** Most candidates were confident with the binomial	condone ≤ instead of <

					expansion and were able to change $(9-16x)^{\frac{3}{2}}$ into a suitable form for expansion. Common errors in the expansion included careless simplification of the x^2 term (often because of cramped writing) and multiplication of this expansion by 9 instead of by 27. A significant number of candidates completely ignored the request for the validity.	Binomial Expansion
			Total	5		
2		i i i	$(1-x)^{-3} = 1 + -3.-x + \frac{-3.-4}{2}(-x)^2 + \ldots$ oe; accept $3x$ for $-3.-x$ & / or $-x^2$ or $(x)^2$ for $(-x)^2$ multiplication by x to produce **AG** (**A**nswer **G**iven)	M1 A1	As result is given, this expansion must be shown and then simplified. It must not just be stated as $1 + 3x + 6x^2 + \ldots$ **Examiner's Comments** The required relationship had been given so, as with all such similar questions, the solutions were examined closely to see, firstly, if the method of expansion was known and, secondly, how accurately it was carried out. Any slight error in accuracy was penalised.	For alternative methods such as expanding $(1-x)^3$ and multiplying by $x + 3x^2 + 6x^3$ or using long division, consult TL
		ii ii ii	Clear indication that $x = 0.1$ is to be substituted (estimated value is) $0.1 + 3(0.1)^2 + 6(0.1)^3 = 0.136$	M1 A1	e.g. $0.1 + 3(0.1)^2 + 6(0.1)^3$ stated **Examiner's Comments** It was not immediately obvious just what the suitable value of x was, but a fair number obtained $x = 0.1$ and substituted into the given expansion.	Calculator value → M0 (0.13717… is calculator value of $\frac{100}{729}$)

		iii	Sight of $1-x=x\left(\frac{1}{x}-1\right)$ or $1-x=-x\left(1-\frac{1}{x}\right)$ or $\left(\frac{1}{x}-1\right)^3=-\left(1-\frac{1}{x}\right)^3$ or $(\frac{1}{x}-1)^{-3}=-\left(1-\frac{1}{x}\right)^{-3}$ or $\left(\frac{1}{x}-1\right)^{-3}=-\left(1-\frac{1}{x}\right)^{-3}$ or equivalent	B1		
		iii	Complete satisfactory explanation (no reference to style) www	B1	(Answer Given)	
		iii	$[1+(-3)(-\frac{1}{x})+\frac{(-3)(-4)}{2}\left(-\frac{1}{x}\right)^2+...]$	M1	Simplified expansion may be quoted - it may have come from result in part (i). Answer for this expansion is not **AG**.	
		iii	$\rightarrow -\frac{1}{x^2}-\frac{3}{x^3}-\frac{6}{x^4}$	A1		
		iii			**Examiner's Comments** The required given identity was not difficult to prove, but most made heavy weather of it; simple stages, showing how $\frac{1}{x}$ and the negative sign were produced, were needed. Whether	

					the identity had or had not been proved, it could then be used to produce the required expansion.	Binomial Expansion
		iv iv iv	Must say “Not suitable” and one of following: Either: requires $\left\lvert\frac{1}{x}\right\rvert < 1$, which is not true if $x = 0.1$ Or: substitution of positive / small value of x in the expansion gives a negative / large value (which cannot be an approximation to 100/729).	B1	This B1 is dep on $x = 0.1$ used in (ii). Or “because $\frac{1}{x} > 1$” Or “it gives - 63100” **Examiner's Comments** A simple explanation was required indicating why the value of x used in part (ii) would be unsuitable if used with the expansion in part (iii). Accepted reasons included the fact that –63100 would be the result or that the substitution of a positive/small value of x would give a negative/large value, which could not be an approximation to $\frac{100}{729}$.	Realistic reason If choice given, do not ignore incorrect comments, but ignore irrelevant / unhelpful ones
			Total	9		
3		i i	$1+(-\frac{1}{2})(-2x)+(-\frac{1}{2})(\frac{-3}{2})\frac{(\pm 2x)^2}{2!}[+...]$ $1+x+\frac{3}{2}x^2$ oe	B1 B1 B1	first two terms third term	allow recovery from omission of brackets do not allow $2x^2$ unless fully recovered in answer

		ii	use of $(x+3)\times \text{their}(1+x+\frac{3}{2}x^2)$	M1	or **B2** www in either part	
		ii	coefficient is 5.5 oe	A1	<u>Examiner's Comments</u> This was very well done by nearly all candidates. A few candidates made a sign error with the first term, and some omitted "2" in part (i). Most gained at least the method mark in part (ii), although a few tried division instead of multiplication.	may be embedded (eg $5.5x^2$ alone or in expansion)
			Total	5		
4		i	$8^{2/3}=4$	B1		may be embedded
		i	$(1-\frac{9x}{8})^{2/3}$ seen	M1	$8^{2/3}+(\frac{2}{3})8^{-1/3}(\pm 9x)+\frac{\frac{2}{3}\times(\frac{2}{3}-1)}{2!}8^{-4/3}(\pm 9x)^2$	ignore extra terms
		i	$1+\left(\frac{2}{3}\right)\left(\frac{\pm 9x}{k}\right)+\frac{1}{2!}\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)\left(\frac{\pm 9x}{k}\right)^2$ where k is an integer greater than 1	M1	$4+(\frac{2}{3})(\frac{1}{2})(\pm 9x)+\frac{\frac{2}{3}\times(\frac{2}{3}-1)}{2!}(\frac{1}{16})(\pm 9x)^2$	or better
		i	$4-3x-\frac{9}{16}x^2$ or $4(1-\frac{3}{4}x-\frac{9}{64}x^2)$ cao	A1	<u>Examiner's Comments</u> This was very well-done, with most candidates scoring at least three out of four marks. A few had difficulty dealing with $8^{2/3}$ and some made sign errors.	

		ii	$-\frac{8}{9} < x < \frac{8}{9}$ or $\lvert x \rvert < \frac{8}{9}$ isw cao	B1	Examiner's Comments Many careless slips were seen, such as $x < \frac{8}{9}$ and $\lvert x \rvert < -\frac{8}{9}$ but the correct answer was seen in the full range of scripts.	Binomial Expansion
			Total	5		
5		a	Obtain $1+\frac{1}{3}px$ $\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)(px)^2$ Obtain $1+\frac{1}{3}px-\frac{1}{9}p^2x^2$	B1(AO 1.1) M1(AO1.1) A1(AO1.1) [3]	Must be simplified | Attempt the x^2 term at least in the form ${}^6C_2kx^2$	
		b	$(1+qx)\left(1+\frac{1}{3}px-\frac{1}{9}p^2x^2\right)$ $=1+\left(\frac{1}{3}p+q\right)x+\left(\frac{1}{3}pq-\frac{1}{9}p^2\right)x^2$ $\frac{1}{3}p+q=1$ (*) $\frac{1}{3}pq-\frac{1}{9}p^2=-\frac{2}{9}$ $2p^2-3p-2=0$	M1(AO 3.1a) M1(AO3.1a) M1(AO1.1) A1(AO1.1) A1FT(AO1.1)	Obtain two equations in p and q and show evidence of substitution for p or q to obtain an | Expand $(1+qx)$ and their $1+\frac{1}{3}px-\frac{1}{9}p^2x^2$ and compare coefficients Or $18q^2-27q+7$	

			$p = 2$ or $-\frac{1}{2}$ $q = \frac{1}{3}$ or $\frac{7}{6}$	[5]	equation in one variable Solve a 3 term quadratic equation in a single variable. Obtain any two values Obtain all 4 values, or FT their *p* and (*)	= 0 Solve their quadratic with indication of correct pairings	Binomial Expansion
			Total	8			
6		a	$1 + x - \frac{1}{2}x^2$	B1(AO1.1) M1(AO1.1a) A1(AO1.1) [3]	Obtain 1 + *x* Attempt third term Obtain correct third term	Terms must be simplified for B / A marks	
		b	$\sqrt{1.08} \approx 1 + 0.04 - 0.5 \times 0.04^2$ $\sqrt{0.36 \times 3} \approx 1.0392$	M1(AO2.1) M1(AO3.1a)	Substitute 0.04 throughout	Need $\sqrt{1.08}$ as well	

			$0.6\sqrt{3} \approx 1.0392$ $\sqrt{3} \approx 1.73$	A1(AO1.1) [3]	Rearrange $\sqrt{1.08}$ to $k\sqrt{3}$ Obtain 1.73 or better | Must see method	Binomial Expansion
		c	Expansion is only valid for $\lvert x \rvert < \frac{1}{2}$	E1(AO2.3) [1]	Explanation must be specific	
			Total	7		
7		i	$1 + 2x$ $\left(\frac{1}{4}\right) \times \left(-\frac{3}{4}\right) \times \frac{(8x)^2}{2!}$ oe soi $1 + 2x - 6x^2$ cao	B1 M1 A1 [3]	allow bracket error | if **M0 allow SC1** for $1 + 4x - 8x^2$ ignore extra terms **Examiner's Comments** This proved accessible to nearly all candidates, with most scoring full marks. A few made arithmetical slips, and a few used the wrong index: –4 and ½ were the usual errors.	

		ii	valid for $\lvert x\rvert < \frac{1}{8}$ oe	B1 [1]	**Examiner's Comments** Although largely well done, this did cause some problems. Common mistakes were to omit one tail of the inequality or to write $x < \left\lvert\frac{1}{8}\right\rvert$. Occasionally $\lvert x \rvert < 8$ was seen.	
			Total	4		
8		a	$(1-\frac{1}{4}x)^{-\frac{1}{2}} = 1 + (-\frac{1}{2})(-\frac{1}{4}x) + (\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(-\frac{1}{4}x)^2$ $= 1 + \frac{1}{8}x + \frac{3}{128}x^2$ $(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}(1-\frac{1}{4}x)^{-\frac{1}{2}} = \frac{1}{2}(1-\frac{1}{4}x)^{-\frac{1}{2}}$ $(4-x)^{-\frac{1}{2}} = \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$	B1 (AO 1.1) M1 (AO 1.1a) A1 (AO 1.1) B1ft (AO 1.1)	Obtain correct first two terms — Allow unsimplified coeffs Attempt third term in expansion of $(1-\frac{1}{4}x)^{-\frac{1}{2}}$ — Product of attempt at binomial coefficient and $(-\frac{1}{4}x)^2$ Allow BOD on missing brackets Allow BOD on missing negative sign in third term Correct third term Correct — Allow unsimplified	

				[4]	<table><tr><td>expansion of $(4-x)^{-\frac{1}{2}}$</td><td>coeffs ft as $\frac{1}{2}$(their three term expansion) No ISW if expression subsequently spoiled by attempt to simplify eg × 256</td></tr></table> Examiner's Comments This question was well answered with many fully correct solutions seen. The most successful solutions made effective use of brackets to ensure that the relevant indices were applied to the entire term including the coefficient of *x*. Showing clear working allowed partial credit to be credited for attempting a valid method should the final answer not be correct.	
		b	$\frac{1}{2}$ $a = 16$ and hence $a = 32$ $2+\frac{1}{2}b=-1$ OR $\frac{1}{16}a+\frac{1}{2}b=-1$	B1ft (AO 3.1a) M1 (AO 2.2a)	<table><tr><td>Correct value of *a* Attempt equation involving *b* and *a*, or their numerical *a*</td><td>ft their first term from (a) Must be using two relevant products (and no others) equated to –1 Allow for</td></tr></table>	

			$b = -6$	A1 (AO 1.1) [3]	<table><tr><td></td><td>attempting equation – do not need to actually attempt solution for M1 Allow BOD if muddle between terms and coefficients</td></tr><tr><td>Solve to obtain $b = -6$</td><td>A0 for $-6x$ unless subsequently corrected</td></tr></table> **Examiner's Comments** This part of the question was also very well answered. Most candidates appreciated the connection with the previous part of the question, and equated coefficients to find the values of *a* and *b*. An alternative method was to cross-multiply and then square both sides before equating coefficients, and this was usually equally successful.	
			Total	7		
9		a	${}^{10}C_4 \times 3^4 \times (-2)^6$ oe $= 1088640$	M1 (AO1.1a) A1 (AO1.1) [2]	<table><tr><td>BC</td><td></td></tr></table>	

		b	$\frac{n!}{(n-7)!7!}2^7 = \frac{n!}{(n-8)!8!}2^8$ 8 = (*n* – 7) × 2 *n* = 11	M1 (AO3.1a) M1 (AO1.1a) A1 (AO1.1) [3]	Correctly obtaining at least two of 8, (*n* – 7) and 2	
		c	$\frac{1}{2}\left(1+\frac{x}{4}\right)^{-0.5}$ $\frac{1}{2}\times\frac{(-0.5)(-1.5)(-2.5)}{3!}\times\left(\frac{1}{4}\right)^3$ $= -\frac{5}{2048}$ oe or –0.00244 (3 sf)	M1 (AO1.1a) M1 (AO1.1) A1FT (AO1.1) A1 (AO1.1) [4]	M1 for $k(1+\frac{x}{4})^{-0.5}$ M1 for $\frac{1}{2}$ follow through their '*k*' cao **BC**	
			Total	9		
10		a	$(1+2x)^{\frac{1}{2}} = 1 + x$ $+\frac{(\frac{1}{2})(-\frac{1}{2})(2x)^2}{2}+\frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(2x)^3}{6}$	B1 (AO 1.1) M1 (AO 1.1a) A1(AO 1.1) A1 (AO 1.1) [4]	Obtain correct first two terms Attempt at least one more term	Must be simplified

			$= 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3$		Obtain correct third term Obtain correct fourth term	Must be simplified Must be simplified	
		b	$(1 + 9x^2)^{-1} = 1 - 9x^2$ $(1 - 9x^2)(1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3)$ $= 1 + x - \frac{19}{2}x^2 - \frac{17}{2}x^3$	B1 (AO 3.1a) M1 (AO 1.1a) A1FT (AO 1.1) [3]	Correct expansion soi Attempt expansion Obtain correct expansion	 FT their **(i)** – must be 4 terms	
		c	$(1 + 2x)^{\frac{1}{2}} \Rightarrow \|x\| < \frac{1}{2}$ $(1 + 9x^2)^{-1} \Rightarrow \|x\| < \frac{1}{3}$ hence $\|x\| < \frac{1}{3}$	M1 (AO 1.1) A1 (AO 2.3) [2]	At least one correct condition seen Correct conclusion, from both correct conditions	oe oe	
			Total	9			