1. For each of the following sequences, state with a reason whether it is convergent, periodic or neither. Each sequence continues in the pattern established by the given terms.

i. $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots$

[1]

ii. 3, 7, 11, 15, …

[1]

iii. 3, 5, –3, –5, 3, 5, –3, –5, …

[1]

2.

i. Find $\sum_{r=1}^{5} \frac{21}{r+2}$

[2]

ii. A sequence is defined by

$$u_1 = a, \text{ where } a \text{ is an unknown constant,}$$
$$u_{n+1} = u_n + 5.$$

Find, in terms of a, the tenth term and the sum of the first ten terms of this sequence.

[3]

3. A sequence is defined by $u_1 = 2$ and $u_{k+1} = \frac{10}{u_k^2}$.

Calculate $\sum_{k=1}^{4} u_k$.

[3]

4. A sequence is defined as follows.
$u_1 = a$, where $a > 0$

To obtain u_{r+1}

- find the remainder when u_r is divided by 3,
- multiply the remainder by 5,
- the result is u_{r+1}.

Find $\sum_{r=2}^{4} u_r$ in each of the following cases.

i. $a = 5$
ii. $a = 6$

[3]

5. Show that $\sum_{r=1}^{4} \ln \frac{r}{r+1} = -\ln 5$. [3]

6. State, with a reason, whether each of the following sequences are convergent, divergent, periodic, or none of these.

(a) $u_n = \sin\left(\frac{n\pi}{2}\right),\ n = 1, 2, 3, \ldots$ [1]

(b) 100, 150, 175, 187.5, 193.75, 196.875, … [1]

(c) $u_{k+1} = (-2)^k u_k$, $u_1 = 3$ [1]

7. (See Insert for H640/03, Practice 4.)
Assume that the length of each side of the equilateral triangle shown in Fig. C1.1 is one unit.

(a) Find the perimeter of the second iteration, shown in Fig. C1.3. [1]

(b) Find an expression for the perimeter of the nth iteration. [3]

(c) Show that, as stated in line 11, the perimeter of the Koch snowflake is not finite. [1]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	converging + valid reason	1		eg converges to 0, $r = ½$, difference between terms decreasing, sum of terms converges to 6, G.P. with $\lvert r\rvert < 1$
		ii	neither + valid reason	1		eg divergent oe, A.P., $d = 4$ oe, convergent and periodic ruled out with correct reasons
		iii	periodic + valid reason	1	**Examiner's Comments** A considerable number of candidates ignored the request to state a reason, and therefore failed to score. Some simply wrote out the first few terms of each sequence and others made comments which were too vague to be credited, such as "decreasing, so converging". A few lost the mark in one or more parts because there was no statement of "convergent" or "neither" — even if a correct reason had been identified.	eg repeating cycle of terms
			Total	3		
2		i	$21\left(\frac{1}{1+2}+\frac{1}{2+2}+\frac{1}{3+2}+\frac{1}{4+2}+\frac{1}{5+2}\right)$ oe soi	M1	may be implied by correct answer	NB 7 + 5.25 + 4.2 + 3.5 + 3 M0 if extra terms or terms missing
		i	22.95 or $\frac{459}{20}$ or $22\frac{19}{20}$	A1	**Examiner's Comments** This was done very well. A few candidates didn't appreciate the meaning of ∑ and merely listed the	

					terms. Similarly, a small number of candidates simply added the first and the last terms. Very few resorted to AP or GP formulae.	
		ii	*a* + 45 cao	B1	mark the final answer must be explicitly stated	
		ii	$\frac{10}{2}(a + a + their 45)$	M1	or $\frac{10}{2}(2a + (10-1) \times 5)$ ignore further work attempting to find *a*	condone wrongly attributed answers
		ii	5 (2a + 45) or 10*a* + 225 cao isw	A1	**Examiner's Comments** Most recognised the arithmetic progression, but some were uncomfortable with a non-numerical a and made a spurious attempt to find its value. For a significant number of candidates, the tenth term was either left as *a* + 9×5 or simplified thus: *a* + 45 = 45a. In both cases an easy mark was lost. Many started again to find the sum of the first ten terms, and did so successfully. There was no credit for those candidates who left their answers in terms of *a* and *d*. A number of candidates wasted time by trying to find the numerical value of *a*.	B2 if correct answer derived from adding terms separately
			Total	5		
3			$u_2 = \frac{10}{2^2}, u_3 = \frac{10}{\text{their } 2.5^2}, u_4 = \frac{10}{\text{their } 1.6^2}$ isw	M1*		NB 2.5, 1.6, 3.90625 or $\frac{10}{4}, \frac{8}{5}, \frac{125}{32}$
			2 + u_2 + u_3 + u_4 soi	M1dep*	must be the sum of 4 terms only	may be implied by eg sight of 3.9 and answer of 10.0
			10.00625 or $\frac{1601}{160}$ or $10\frac{1}{80}$ cao isw	A1	**B3** if unsupported	NB 2.5, 1.1, 0. 625 scores **M0M0**

					Examiner's Comments A little under half of candidates achieved full marks on this question. Approximately 20% prematurely rounded their answers and lost the final accuracy mark, and a few found the sum of the second to fifth terms inclusive instead of the first to fourth. The most common error for those who failed to score at all was to treat the sequence as being defined algebraically, but a few candidates misused the formula for the sum of an arithmetic or geometric progression.	
			Total	3		
4			(i) [5], 10, 5, [10] [10 + 5 + 10 =] 25 (ii) 0	M1 A1 B1	ignore extra terms not from wrong working **Examiner's Comments** Many candidates had difficulty with this question. In some cases it would seem that this was due to a failure to read the question properly, but it was also apparent that a significant minority did not understand how to generate the terms of the sequence. Even many of those who did generate the terms successfully then either ignored the sigma notation or summed an incorrect number of terms.	condone wrongly attributed terms **B2** for 25 unsupported
			Total	3		
5			$\sum_{r=1}^{4} \ln \frac{r}{r+1} = \ln \frac{1}{2} + \ln \frac{2}{3} + \ln \frac{3}{4} + \ln \frac{4}{5}$ = ln 1 − ln 2 + ln 2 − ln 3 + ln 3 − ln 4 + ln 4 − ln 5	B1(AO1.1) M1(AO3.1a)	soi use of $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$	

			$= 0 - \cancel{\ln 2} + \cancel{\ln 2} - \cancel{\ln 3} + \cancel{\ln 3} - \cancel{\ln 4} + \cancel{\ln 4} - \ln 5$ $= -\ln 5$	E1(AO2.2a) [3]	or $\ln a + \ln b = \ln ab$ $\ln\left(\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\right) = \ln\frac{1}{5}$ $= \ln 1 - \ln 5 = -\ln 5$ AG	
			Total	3		
6		a	periodic since 1, 0, 1, 0, repeats	B1 (AO1.2) [1]		
		b	convergent, since difference between terms is decreasing	B1 (AO2.4) [1]		
		c	divergent, since magnitude of difference between terms increasing	B1 (AO1.2) [1]	allow "difference between terms increasing"	
			Total	3		
7		a	$\frac{16}{3}$ [units]	B1 (AO 2.2a) [1]	oe	
		b	At each iteration, the perimeter is $\frac{4}{3}$ times the previous perimeter Perimeter of *n*th iteration $= 3\times\left(\frac{4}{3}\right)^n$	M1 (AO 3.1a) A2 (AO 1.1, 2.2a) [1]	May be a specific instance Allow **A1** for 3	

					times a power of $\frac{4}{3}$	
		c	As $n \to \infty,\ 3\times\left(\frac{4}{3}\right)^n \to \infty$	E1 (AO 2.4) [1]	or $3\times\left(\frac{4}{3}\right)^n$ increases without limit as n increases	
			Total	5		