

1. For each of the following sequences, state with a reason whether it is convergent, periodic or neither. Each sequence continues in the pattern established by the given terms.

i. $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \dots$

[1]

ii. $3, 7, 11, 15, \dots$

[1]

iii. $3, 5, -3, -5, 3, 5, -3, -5, \dots$

[1]

2.

i. Find $\sum_{r=1}^5 \frac{21}{r+2}$

[2]

- ii. A sequence is defined by

$$u_1 = a, \text{ where } a \text{ is an unknown constant,}$$
$$u_{n+1} = u_n + 5.$$

Find, in terms of a , the tenth term and the sum of the first ten terms of this sequence.

[3]

3.

A sequence is defined by $u_1 = 2$ and $u_{k+1} = \frac{10}{u_k^2}$.

Calculate $\sum_{k=1}^4 u_k$.

[3]

4. A sequence is defined as follows.

$$u_1 = a, \text{ where } a > 0$$

To obtain u_{r+1}

- find the remainder when u_r is divided by 3,
- multiply the remainder by 5,
- the result is u_{r+1} .

Find $\sum_{r=2}^4 u_r$ in each of the following cases.

i. $a = 5$

ii. $a = 6$

[3]

5. Show that $\sum_{r=1}^4 \ln \frac{r}{r+1} = -\ln 5$. [3]

6. State, with a reason, whether each of the following sequences are convergent, divergent, periodic, or none of these.

(a) $u_n = \sin\left(\frac{n\pi}{2}\right), n = 1, 2, 3, \dots$ [1]

(b) 100, 150, 175, 187.5, 193.75, 196.875, ... [1]

(c) $u_{k+1} = (-2)^k u_k, u_1 = 3$ [1]

7. (See Insert for H640/03, Practice 4.)
Assume that the length of each side of the equilateral triangle shown in Fig. C1.1 is one unit.
- (a) Find the perimeter of the second iteration, shown in Fig. C1.3. [1]
- (b) Find an expression for the perimeter of the n th iteration. [3]
- (c) Show that, as stated in line 11, the perimeter of the Koch snowflake is not finite. [1]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	converging + valid reason	1		eg converges to 0, $r = \frac{1}{2}$, difference between terms decreasing, sum of terms converges to 6, G.P. with $ r < 1$
		ii	neither + valid reason	1		eg divergent oe, A.P., $d = 4$ oe, convergent and periodic ruled out with correct reasons
		iii	periodic + valid reason	1	<p>Examiner's Comments</p> <p>A considerable number of candidates ignored the request to state a reason, and therefore failed to score. Some simply wrote out the first few terms of each sequence and others made comments which were too vague to be credited, such as "decreasing, so converging". A few lost the mark in one or more parts because there was no statement of "convergent" or "neither" — even if a correct reason had been identified.</p>	eg repeating cycle of terms
			Total	3		
2		i	$21 \left(\frac{1}{1+2} + \frac{1}{2+2} + \frac{1}{3+2} + \frac{1}{4+2} + \frac{1}{5+2} \right)$ oe soi	M1	may be implied by correct answer	NB $7 + 5.25 + 4.2 + 3.5 + 3$ M0 if extra terms or terms missing
		i	$22.95 \text{ or } \frac{459}{20} \text{ or } 22\frac{19}{20}$	A1	<p>Examiner's Comments</p> <p>This was done very well. A few candidates didn't appreciate the meaning of \sum and merely listed the</p>	

					terms. Similarly, a small number of candidates simply added the first and the last terms. Very few resorted to AP or GP formulae.	
		ii	$a + 45$ cao	B1	mark the final answer must be explicitly stated	
		ii	$\frac{10}{2}(a + a + \text{their } 45)$	M1	or $\frac{10}{2}(2a + (10 - 1) \times 5)$	condone wrongly attributed answers
		ii	$5(2a + 45)$ or $10a + 225$ cao isw	A1	ignore further work attempting to find a Examiner's Comments Most recognised the arithmetic progression, but some were uncomfortable with a non-numerical a and made a spurious attempt to find its value. For a significant number of candidates, the tenth term was either left as $a + 9 \times 5$ or simplified thus: $a + 45 = 45a$. In both cases an easy mark was lost. Many started again to find the sum of the first ten terms, and did so successfully. There was no credit for those candidates who left their answers in terms of a and d . A number of candidates wasted time by trying to find the numerical value of a .	B2 if correct answer derived from adding terms separately
			Total	5		
3			$u_2 = \frac{10}{2^2}, u_3 = \frac{10}{\text{their } 2.5^2}, u_4 = \frac{10}{\text{their } 1.6^2}$ isw $2 + u_2 + u_3 + u_4$ soi $\frac{1601}{160}$ or $10\frac{1}{80}$ cao isw 10.00625 or	M1* M1dep* A1	must be the sum of 4 terms only B3 if unsupported	NB 2.5, 1.6, 3.90625 $\frac{10}{4}, \frac{8}{5}, \frac{125}{32}$ or may be implied by eg sight of 3.9 and answer of 10.0 NB 2.5, 1.1, 0. 625 scores MOMO

			$= 0 - \ln 2 + \ln 2 - \ln 3 + \ln 3 - \ln 4 + \ln 4 - \ln 5$ $= -\ln 5$	E1(AO2.2a) [3] AG	$\text{or } \ln a + \ln b = \ln ab$ $\ln \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \right) = \ln \frac{1}{5}$ $= \ln 1 - \ln 5 = -\ln 5$	
			Total	3		
6		a	periodic since 1, 0, 1, 0, repeats	B1 (AO1.2) [1]	<input type="text"/>	
		b	convergent, since difference between terms is decreasing	B1 (AO2.4) [1]	<input type="text"/>	
		c	divergent, since magnitude of difference between terms increasing	B1 (AO1.2) [1]	allow "difference between terms increasing"	
			Total	3		
7		a	<input type="text" value="16/3"/> [units]	B1 (AO 2.2a) [1]	oe <input type="text"/>	
		b	<input type="text" value="At each iteration, the perimeter is"/> <input type="text" value="4/3"/> times the previous perimeter <input type="text" value="Perimeter of n-th iteration"/> = <input type="text" value="3 * (4/3)^n"/>	M1 (AO 3.1a) A2 (AO 1.1, 2.2a) [1]	<input type="text"/> May be a specific instance Allow A1 for 3	

					times a power of $\frac{4}{3}$	
		c	As $n \rightarrow \infty$, $3 \times \left(\frac{4}{3}\right)^n \rightarrow \infty$	E1 (AO 2.4) [1]	or $3 \times \left(\frac{4}{3}\right)^n$ increases without limit as n increases	
			Total	5		