1. The nth term of a sequence, u_n, is given by

$$u_n = 12 - \tfrac{1}{2}n.$$

i. Write down the values of u_1, u_2 and u_3. State what type of sequence this is.

[2]

ii. Find $\sum_{n=1}^{30} u_n$.

[3]

2.

i. Find $\sum_{r=1}^{5} \frac{21}{r+2}$

[2]

ii. A sequence is defined by

$$u_1 = a, \text{ where } a \text{ is an unknown constant,}$$
$$u_{n+1} = u_n + 5.$$

Find, in terms of a, the tenth term and the sum of the first ten terms of this sequence.

[3]

3. An arithmetic progression has tenth term 11.1 and fiftiethterm 7.1. Find the first term and the common difference. Find also the sum of the first fifty terms of the progression.

[5]

4.

(i) Calculate $\sum_{r=1}^{5}(3r+2)$. [2]

(ii) An arithmetic progression (AP) has first term 4.2 and sixth term 1.8. Find the common difference of this AP. [2]

5. A firm takes on two new employees, Arif and Bettina.

- Arif starts on an annual salary of £30 000, and his salary increases by £1000 each year after that.
- Bettina starts on an annual salary of £25 000, and her salary then increases by 5% each year after that. (So, for example, Bettina's salary in year 3 is 5% greater than her salary in year 2.)

(i) Show that Arif earns more than Bettina in year 10 of their employment, but Arif earns less than Bettina in year 11. [4]

(ii) Show that the total amounts earned by each of Arif and Bettina during their employment up to the end of year 17, correct to the nearest £100, are equal. [4]

At the end of year *n*, the total that Bettina has earned during this employment is greater than £*M*.

(iii) Show that $$n > \frac{\log_{10}(M + 500\,000) - \log_{10} 500\,000}{\log_{10} 1.05}$$

Hence find in which year the total that Bettina has earned during this employment is first greater than
£1.2 million. [5]

6. An arithmetic sequence has third term 6 and ninth term 30. Find the sum of the first 100 terms. [4]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	11.5, 11 and 10.5 oe	B1		ignore labelling
		i	arithmetic and / or divergent	B1	allow AP ignore references to *a*, *d* or *n* Examiner's Comments Nearly all candidates spotted the algebraic definition and correctly found the required terms. A few lost a mark by calculating the first, second and fourth term, and a few thought it was an inductive definition and substituted $u1$ in the formula instead of $n = 2$. The most common description was "arithmetic"; a few candidates also earned the mark with "divergent". However, a significant minority either omitted a description altogether, gave an incorrect answer (most commonly "convergent" or "geometric" and occasionally "periodic") or spoiled their correct answer by hedging their bets: for example, "converging arithmetic" was fairly common.	incorrect embellishments such as converging arithmetic…, diverging geometric… do not score. **B0** if a choice is given e.g. AP / GP.
		ii	$n = 30$ identified as number of terms in relevant AP	B1		e.g. 1 + 2 + 3 + … + 30 is not a relevant AP
		ii	$S_{30} = \frac{30}{2}(2\times 11.5 + (30-1)\times -0.5)$	M1	or $S_{30} = \frac{30}{2}(11.5 + -3)$ allow recovery from slip in working (e.g. omission of minus sign)	condone one error in *a, d* or *n* but do not condone $l = -½$
		ii	127.5 oe	A1	Examiner's Comments A little over half of candidates scored full marks on this question. A surprising number either specifically identified d as ½, or omitted the minus sign when calculating the sum of the A.P., and ended up with an answer of 562.5. Very few of these candidates had the	**SC3** if each term calculated and summed to correct answer or for 127.5 unsupported

					sense that something must have gone wrong. A few others mistakenly identified a as 12, but were still able to score 2 marks. Some candidates did not use the formula, instead writing out all the terms and calculating the sum directly: as often as not the arithmetic went astray and so only the first mark was earned. Approximately one fifth of candidates made no headway. The sigma notation proved insurmountable for a few, and others used the formula for the sum of a geometric progression or simply attempted to find the *n*th term. Others confused Σu_n with Σn, and thus failed to score.	
			Total	5		
2		i	$21\left(\frac{1}{1+2}+\frac{1}{2+2}+\frac{1}{3+2}+\frac{1}{4+2}+\frac{1}{5+2}\right)$ oe soi	M1	may be implied by correct answer	NB 7 + 5.25 + 4.2 + 3.5 + 3 M0 if extra terms or terms missing
		i	22.95 or $\frac{459}{20}$ or $22\frac{19}{20}$	A1	**Examiner's Comments** This was done very well. A few candidates didn't appreciate the meaning of ∑ and merely listed the terms. Similarly, a small number of candidates simply added the first and the last terms. Very few resorted to AP or GP formulae.	
		ii	*a* + 45 cao	B1	mark the final answer must be explicitly stated	
		ii	$\frac{10}{2}(a+a+their45)$	M1	or $\frac{10}{2}(2a+(10-1)\times 5)$ ignore further work attempting to find *a*	condone wrongly attributed answers
		ii	5 (2a + 45) or 10*a* + 225 cao isw	A1	**Examiner's Comments** Most recognised the arithmetic progression, but some were uncomfortable with a non-numerical a and made a spurious attempt to find its value. For a significant number of candidates,	B2 if correct answer derived from adding terms separately

					the tenth term was either left as $a + 9\times5$ or simplified thus: $a + 45 = 45a$. In both cases an easy mark was lost. Many started again to find the sum of the first ten terms, and did so successfully. There was no credit for those candidates who left their answers in terms of *a* and *d*. A number of candidates wasted time by trying to find the numerical value of *a*.	
			Total	5		
3			$a + (10 - 1)d = 11.1$ and $a + (50 - 1)d = 7.1$	M1	may be implied by $40d = \pm4$ or embedded in attempt to solve	condone one slip in coefficient of *d*
			$d = -0.1$	A1	if unsupported, **B2** for one of these and **B3** for both	
			$a = 12$	A1		
			$\frac{1}{2}\times 50(\textit{their}\ a + 7.1)$ with $a > 11.1$	M1	or $\frac{50}{2}(2a+(50-1)d)$ with $a > 11.1$ and $d < 0$	
			477.5 or 477½ or $\frac{955}{2}$ cao	A1	**Examiner's Comments** Most candidates knew what to do here, but a surprisingly high number misread "fiftieth" as "fifteenth", and a few misread "fiftieth" as "fifth". A few then also misread one of the numbers. However, most read fifty correctly. The majority went on to solve their equations successfully, but a surprising number obtained a positive value for *d* and simply carried on, without stopping to think that this could not possibly be correct. Candidates would to well to ask themselves whether or not their answer is sensible in the context of the original question. It seemed that many candidates simply didn't see the request to find the sum of the first fiftyterms, and stopped after finding *a* and *d*.	if **M0**, **B2** for any form of correct answer www
			Total	5		

4		i	$3\times1 + 2 + 3\times2 + 2 + 3\times3 + 2 + 3\times4 + 2 + 3\times5 + 2$ oe soi 55	B1 B1 [2]	or $3\times\frac{1}{2}\times5\times(5+1)+2\times5$; or $\frac{5}{2}\left[2\times5+(5-1)\times3\right]$ **B2** for 55 unsupported **Examiner's Comments** This was done very well. A small minority of candidates failed to score, usually through misusing formulae associated with arithmetic or geometric progressions. A small number of candidates demonstrated the correct method, but slipped up with arithmetic.	
4		ii	$4.2 + 5d = 1.8$ soi -0.48 or $-\frac{12}{25}$ isw	M1 A1 [2]	or $(1.8 - 4.2) \div 5$ oe ; **M0** for $(4.2 - 1.8) \div 5$ if not recovered **B2** for correct answer unsupported **Examiner's Comments** This was done very well, too. However, some candidates failed to appreciate that d had to be negative, and a few interchanged a and d.	

			Total	4		
5		i	[year 10] A : 39 000 B : 38 783.205…isw r.o.t. to 6 or more significant figures [year 11] A : 40 000 B : 40 722.365…isw r.o.t. to 6 or more significant figures	B1 B1 B1 B1 [4]	or 38 800 or 38 780 or 38 783 or 40 700 or 40 720 or 40 722 **Examiner's Comments** The majority of candidates gained full marks on this question. A few candidates listed all the terms and lost accuracy on the way, and a few misused the formulae.	**B0** for any which are wrongly attributed
		ii	A: $\frac{17}{2}(2\times30000+16\times1000)$ or $\frac{17}{2}(30000+46000)$ $= 646\ 000$	M1 A1 M1	if **M0** and **B0** allow **SC1** for 30 000 + 31 000 +…+ 46 000 = 646 000 646 000 unsupported is **M0A0**	if **M0** then **B2** for complete sum written out and correct answer obtained

			B: $\dfrac{25000(1.05^{17}-1)}{1.05-1}$ = 646 009.15…r.o.t. to 6 significant figures or more	A1 [4]	if **M0** and **B0** allow **SC1** for 25 000 + 25 000 × 1.05 + …+ 25 000 × 1.05^{16} = 646 009.15… 646009…unsupported is **M0A0** **A0** for 646 000 only after award of **M1** | if **M0** then **B2** for complete sum written out and correct answer obtained **Examiner's Comments** This part of the question was also very well done, but some candidates did not give enough detail to "show that" Arif and Bettina earned the same amount to the nearest £100. A common mistake was to write down Bettina's earnings as £646 000 without showing the value before rounding.	
		iii	$\dfrac{25000(1.05^n-1)}{1.05-1} > M$ $1.05^n > \dfrac{M+500000}{500000}$ www oe	M1 A1	allow eg $\dfrac{25000(1-1.05^n)}{-0.05} > M$ at least one correct intermediate step to obtain correct inequality with 1.05^n isolated on LHS | condone = or <	

			$\log_{10} 1.05^n > \log_{10}\left(\frac{M+500000}{500000}\right)$ oe eg $n\log_{10} 1.05 > \log_{10}(M+500000) - \log_{10} 500000$ $n > \frac{\log_{10}(M+500000) - \log_{10} 500000}{\log_{10} 1.05}$ www 26 cao *Alternatively* $\frac{25000(1.05^n - 1)}{1.05 - 1} > M$ $\log_{10}(500\,000 \times 1.05^n) > \log_{10}(M + 500\,000)$ oe $\log_{10}(1.05^n) > \log_{10}(M + 500\,000) - \log_{10} 500\,000$ oe $n > \frac{\log_{10}(M+500000) - \log_{10} 500000}{\log_{10} 1.05}$ www 26 cao	A1 A1 B1 M1 A1 A1 A1 B1 [5]	following at least one correct intermediate step NB $n > 25.08\ldots$ following at least one correct intermediate step following at least one correct intermediate step NB $n > 25.08\ldots$	condone omission of brackets on RHS and / or omission of base B0 for $n > 26$	

					B0 for $n > 26$ **Examiner's Comments** A minority of candidates presented clear, concise solutions to derive the inequality, and went on to obtain the correct value of n. Many candidates, however, did not attempt the derivation or started with the final statement. A few went on to obtain the correct value of n, although 25 was a common wrong answer.	
			Total	13		
6			$a + 2d = 6$ $a + 8d = 30$ $d = 4$, $a = -2$ $\frac{100}{2}\left(2\times(-2)+99\times 4\right)$ 19600	B1 (AO 1.1) M1 (AO 3.1a) M1 (AO 1.1) A1 (AO 1.1) [4]	One equation (could be $6d = 24$) BC Use of formula	

			Total	4		