1. $g(x) = (2 + ax)^8$ where a is a constant

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$

(a) find the value of a.

(4)

Using this value of a,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

a)

By the binomial expansion, the x^5 term is :

BINOMIAL SERIES :

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} \times b + \ldots + b^n$$

$$\binom{8}{5} \times 2^{8-5} (ax)^5$$

$$= \frac{8!}{5!(8-5)!} \times 2^3 \times a^5 x^5 \quad (1)$$

where : $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

$$= 448\, a^5 x^5 = 3402\, x^5 \quad (1)$$

$$= 448\, a^5 = 3402$$

$$= a^5 = \frac{3402}{448} = \frac{243}{32} \quad (1)$$

$$a = \frac{\sqrt[5]{243}}{\sqrt[5]{32}}$$

$$a = \frac{3}{2} \quad (1)$$

b)

The first constant is 2^8 (1) (the constant of the expansion $\times 1$) $= 256$.

The second constant is the x^4 term of the expansion $\times \frac{1}{x^4}$ term.

$= {}^8C_4 \times 2^4 a^4 = 70 \times 16 \times \frac{181}{16} = 5670$ (1)

The constant term is $256 + 5670 = 5926$ (1)

2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3-\frac{2x}{9}\right)^8$$

giving each term in simplest form.

(4)

$$32x9=\left(\frac{x-8}{\square x}\right)\left(\square-\frac{\square x}{\square}\right)^{\square}$$

(b) Find the coefficient of x^2 in the series expansion of f(x), giving your answer as a simplified fraction.

(2)

a) $\left(3-\frac{2x}{9}\right)^8 = \binom{8}{0}(3)^8\left(-\frac{2x}{9}\right)^0 + \binom{8}{1}(3)^7\left(-\frac{2x}{9}\right)^1 +$ (1)

1st term 2nd term

$\binom{8}{2}(3)^6\left(-\frac{2x}{9}\right)^2 + \binom{8}{3}(3)^5\left(-\frac{2x}{9}\right)^3 + \ldots$ (1)

3rd term 4th term

$= 6561 - 3888x + 1008x^2 - \frac{448}{3}x^3 + \ldots$ (1)

b) $\left(\frac{x-1}{2x}\right)\left(3-\frac{2x}{9}\right)^8$

$\left(\frac{x}{2x}-\frac{1}{2x}\right)\left(3-\frac{2x}{9}\right)^8$

$\left(\frac{1}{2}-\frac{1}{2x}\right)\left(6561-3888x+1008x^2-\frac{448}{3}x^3+\ldots\right)$

coefficient of x^2: $\left(\frac{1}{2}\times 1008\right)+\left(-\frac{1}{2}\times-\frac{448}{3}\right)$ (1)

$= 504 + \frac{224}{3}$

$= \frac{1736}{3}$ (1)

3. Find, in simplest form, the coefficient of x^5 in the expansion of

$$(5+8x^2)\left(3-\frac{1}{2}x\right)^6$$

(5)

Since the first term is $(5+8x^2)$, we need term of x^5 and x^3 from the second term to find the coefficient of x^5 from the expansion.

$= 5 \times x^5$ term

$: 8x^2 \times x^3$ term

x^5 term $: {}^6C_5 \times 3^1\left(-\frac{1}{2}x\right)^5 = -\frac{9}{16}x^5$ (1)

x^3 term $: {}^6C_3 \times 3^3\left(-\frac{1}{2}x\right)^3 = -\frac{135}{2}x^3$ (1)

coefficient of x^5 in the expansion :

$$\left(5 \times -\frac{9}{16}x^5\right) + \left(8x^2 \times -\frac{135}{2}x^3\right)$$ (1) (1)

$$= -\frac{45}{16}x^5 + (-540x^5)$$

$$= \left(-\frac{45}{16} - 540\right)x^5$$

$$= -\frac{8685}{16}$$ (1)