

Sequences Questions

- 5 The n th term of a sequence is u_n .

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = 200 \quad u_2 = 150 \quad u_3 = 120$$

- (a) Show that $p = 0.6$ and find the value of q . *(5 marks)*
- (b) Find the value of u_4 . *(1 mark)*
- (c) The limit of u_n as n tends to infinity is L . Write down an equation for L and hence find the value of L . *(3 marks)*
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- 3 The first term of an arithmetic series is 1. The common difference of the series is 6.

- (a) Find the tenth term of the series. *(2 marks)*
- (b) The sum of the first n terms of the series is 7400.
- (i) Show that $3n^2 - 2n - 7400 = 0$. *(3 marks)*
- (ii) Find the value of n . *(2 marks)*
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- 4 (a) The expression $(1 - 2x)^4$ can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q . *(3 marks)*

- (b) Find the coefficient of x in the expansion of $(2 + x)^9$. *(2 marks)*
- (c) Find the coefficient of x in the expansion of $(1 - 2x)^4(2 + x)^9$. *(3 marks)*
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5 The second term of a geometric series is 48 and the fourth term is 3.

(a) Show that one possible value for the common ratio, r , of the series is $-\frac{1}{4}$ and state the other value. (4 marks)

(b) In the case when $r = -\frac{1}{4}$, find:

(i) the first term; (1 mark)

(ii) the sum to infinity of the series. (2 marks)

7 (a) The first four terms of the binomial expansion of $(1 + 2x)^8$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the integers a , b and c . (4 marks)

(b) Hence find the coefficient of x^3 in the expansion of $(1 + \frac{1}{2}x)(1 + 2x)^8$. (3 marks)

2 The n th term of a geometric sequence is u_n , where

$$u_n = 3 \times 4^n$$

(a) Find the value of u_1 and show that $u_2 = 48$. (2 marks)

(b) Write down the common ratio of the geometric sequence. (1 mark)

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is $4^k - 4$, where k is an integer. (3 marks)

(ii) Hence find the value of $\sum_{n=2}^{12} u_n$. (1 mark)

4 An arithmetic series has first term a and common difference d .

The sum of the first 29 terms is 1102.

(a) Show that $a + 14d = 38$. (3 marks)

(b) The sum of the second term and the seventh term is 13.

Find the value of a and the value of d . (4 marks)

Sequences Answers

5(a)	$150 = 200p + q$	M1		Either equation
	$120 = 150p + q$	A1		
		m1		Both (condone embedded values for the M1A1)
	$p = 0.6$	A1	5	Valid method to solve two simultaneous eqns in p and q to find either p or q AG (condone if left as a fraction)
	$q = 30$	B1		
(b)	$u_4 = 102$	B1F✓	1	Ft on $(72 + q)$
(c)	$L = pL + q$; $L = 0.6L + 30$	M1		
	$L = \frac{q}{1-p}$	m1		
	$L = 75$	A1F✓	3	Ft on $2.5q$
Total			9	

3(a)	(Tenth term) $= a + (10-1)d$	M1	2	NMS or rep. addn. B2 CAO SC if M0 award B1 for $6n-5$ OE
 $= 1 + 9(6) = 55$	A1		
(b)(i)	$S_n = \frac{n}{2}[2 + (n-1)6]$	M1		Formula for $\{S_n\}$ with either $a = 1$ or $d = 6$ substituted
	$\frac{n}{2}[2 + 6n - 6] = 7400$	A1		Eqn formed with some expansion of brackets
	$3n^2 - 2n = 7400 \Rightarrow 3n^2 - 2n - 7400 = 0$	A1	3	CSO AG
(ii)	$(3n + 148)(n - 50) = 0$	M1		Formula/factorisation OE
	$\Rightarrow n = 50$	A1	2	NMS single ans. 50. B2 CAO NMS 50 and $-49.3(3\dots)$ B1 CAO
Total			7	

4(a)	$(1-2x)^4 = (1)^4 + 4(1)^3(-2x) + 6(1^2)(-2x)^2 + [4(1)(-2x)^3 + (-2x)^4]$ $= [1] - 8x + 24x^2 + [-32x^3 + 16x^4]$	M1	3	Any valid method as far as term(s) in x and term(s) in x^2 . $p = -8$ Accept $-8x$ even within a series. $q = 24$ Accept $24x^2$ even within a series.
		A1		
(b)	x term is $\binom{9}{1} 2^8 x$ Coefficient of x term is $= 9 \times 2^8 = 2304 (=k)$	M1	2	OE Condone 2304x
		A1		
(c)	$(1-2x)^4 (2+x)^9 = (1+px+...)(2^9+kx...)$ $= \dots$ $= \dots + kx + px(2^9) + \dots$ Coefficient of x is $k + 512p$ $= 2304 - 4096 = -1792$	M1	3	Uses (a) and (b) oe (PI) Multiply the two expansions to get x terms ft on candidate's values of k and p . Condone $-1792x$ SC If 0/3 award B1ft for $p+k$ evaluated
		M1		
		A1ft		
Total			8	

5(a)	$ar = 48; \quad ar^3 = 3$ $\Rightarrow 16r^2 = 1$ $r^2 = \frac{1}{16} \Rightarrow r = -\frac{1}{4}$ or $r = \frac{1}{4}$	B1	4	For either. OE Elimination of a OE CSO AG Full valid completion. SC Clear explicit verification (max B2 out of 3.)
		M1		
		A1		
		B1		
(b)(i)	$a = -192$	B1	1	
(ii)	$\frac{a}{1-r} = \frac{a}{1-\left(-\frac{1}{4}\right)}$ $S_{\infty} = \frac{-768}{5} (= -153.6)$	M1	2	$\frac{a}{1-r}$ <u>used</u> Ft on candidate's value for a . i.e. $\frac{4}{5}a$ SC candidate uses $r = 0.25$, gives $a = 192$ and sum to infinity = 256. (max. B0 M1A1)
		A1ft		
Total			7	

7(a)	$(1+2x)^8$ $=1+\binom{8}{1}(2x)^1+\binom{8}{2}(2x)^2+\binom{8}{3}(2x)^3+\dots$ $=1+16x+112x^2+448x^3+\dots$ $\{a=16, b=112, c=448\}$	M1		Any valid method. PI by correct value for a, b or c
		A1A1		A1 for each of a, b, c
		A1	4	
(b)	x^3 terms from expn. of $\left(1+\frac{1}{2}x\right)(1+2x)^8$ are cx^3 and $\frac{1}{2}x(bx^2)$ $cx^3 + \frac{1}{2}x(bx^2)$	M1		Either
		A1		b, c or candidate's values for b and c from (a)
	Coefficient of x^3 is $c+0.5b=504$	A1ft	3	Ft on candidate's $(c+0.5b)$ provided b and c are positive integers >1
Total			7	

2(a)	$u_1 = 12$ $u_2 = 3 \times 4^2 = 48$	B1		
		B1	2	CSO AG (be convinced)
(b)	$r = 4$	B1	1	
(c)(i)	$\{S_{12}\} = \frac{a(1-r^{12})}{1-r}$ $= \frac{12(1-4^{12})}{1-4}$ $= \frac{12(1-4^{12})}{-3} = -4(1-4^{12}) = 4^{13} - 4$	M1		OE Using a correct formula with $n = 12$
		A1ft		Ft on answer for u_1 in (a) and r in (b)
		A1	3	CAO Accept $k = 13$ for 4^{13} term
(ii)	$\sum_{n=2}^{12} u_n = (4^{13} - 4) - u_1$ $= 67108848$	B1	1	
Total			7	

4(a)	$\{S_{29} = \frac{29}{2}[2a + 28d]$	M1		Formula for S_n with $n = 29$ substituted and with a and d
	$29(a + 14d) = 1102$	m1		Equation formed then some manipulation
	$a + 14d = \frac{1102}{29} \Rightarrow a + 14d = 38$	A1	3	CSO AG
(b)	$u_2 = a + d \quad u_7 = a + 6d$	B1		Either expression correct
	$u_2 + u_7 = 13 \Rightarrow 2a + 7d = 13$	M1		Forming equation using u_2 & u_7 , both in form $a + kd$
	e.g. $21d = 63; 3a = -12$	m1		Solving $a + 14d = 38$ with candidate's ' $2a + 7d = 13$ ' to at least stage of elimination of either a or d
	$a = -4 \quad d = 3$	A1	4	Both correct
	Total		7	