1. Either prove or disprove each of the following statements.

i. 'If m and n are consecutive odd numbers, then at least one of m and n is a prime number.'

[2]

ii. ' If m and n are consecutive even numbers, then mn is divisible by 8.'

[2]

2. Suppose x is an irrational number, and y is a rational number, so that $y = \frac{m}{n}$, where m and n are integers and $n \neq 0$. Prove by contradiction that $x + y$ is not rational. [4]

3. You are given that the sum of the interior angles of a polygon with n sides is $180(n - 2)°$. Using this result, or otherwise, prove that the interior angle of a regular polygon cannot be 155°. [3]

4. Fig. 4 shows a cuboid $ABCDEFGH$. You are given that $FA > BC > AB > 0$. The distance between F and C is 5 cm. Given that the lengths AB and BC are both integers, prove that the length FA cannot be an integer. [3]

Fig. 4

5. (See Insert for Jun18 64003.) It is given in lines 31 – 32 that the square has the smallest perimeter of all rectangles with the same area. Using this fact, prove by contradiction that among rectangles of a given perimeter, $4L$, the square with side L has the largest area. [3]

6. Prove that $x^2 + x + 2 > 1$ for all real values of x. [3]

7. Starting from the formula Price elasticity of demand $= \dfrac{\text{\% increase in quantity demanded}}{\text{\% increase in price}}$, as given on line 19, show that, at point A in Fig. C1 the price elasticity of demand is $\dfrac{P}{mQ}$, where m is the gradient of the straight line. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	False e.g. neither 25 and 27 are prime	B1	correct counter–example identified justified correctly **Examiner's Comments** Most candidates stated this was false and looked for a counter-example, usually 25 and 27. We did require them to show their counter-examples were composite for a second B mark.	Need not explicitly say 'false'
		i	as 25 is div by 5 and 27 by 3	B1		
		ii	True: one has factor of 2, the other 4, so product must have factor of 8.	B2	or algebraic proofs: e.g. $2n(2n + 2) = 4n(n + 1) =$ 4 × even × odd no so div by 8 **Examiner's Comments** This was less successful. Most candidates could see it was true, but then failed to come up with a coherent argument. Some wrote $2n(2n+2)= 4n^2 + 4n$ or equivalent, but then failed to explain why this is then divisible by 8 (rather than just 4, which got 1 out of 2). Most successful candidates got the idea that alternate even numbers are divisible by 4 and hence the product of this with another even number is divisible by 8.	B1 for stating with justification div by 4 e.g. both even, or from $4(n^2 + n)$ or $4pq$
			Total	4		
2			Suppose $x + y$ is rational So $x+y=\frac{p}{q}$, where p and q are integers $\Rightarrow\ x=\frac{p}{q}-\frac{m}{n}=\frac{(pn-mq)}{qn}$ which is rational x is irrational so this is a contradiction	E1(AO2.1) B1(AO2.1) B1(AO3.1a) E1(AO2.4) [4]	or stating that the difference of two fractions is rational	
			Total	4		

Question			Answer	Marks	Guidance	
3			Suppose the polygon has n sides. Then $180(n-2) = 155n$ $\Rightarrow 25n = 360$ [$\Rightarrow n = 14.4$] which is impossible as n is an integer So no regular polygon has interior angle 155° **or** When $n = 14$, int angle = 180×12/14 = 154.29° When $n = 15$, int angle = 180×13/15 = 156° So no n which gives an interior angle 155°.	M1 A1 A1cao B1 B1 B1 [3]	or sum of ext angles = 360° so $25n = 360$ or 72/5 clear statement of conclusion accept 154° **Examiner's Comments** Candidates scored full marks or zero marks in roughly equal numbers here. Most gave the first method shown in the mark scheme, namely solving $180(n-2) = 155n$ to get n = 14.4, but we also saw some examples of the second approach, finding the interior angles for 14 and 15 sides. By far the most common error was to solve $180(n-2) = 155$, getting n = 2.86.	
			Total	3		
4			<table><tr><th>*AB*</th><th>*BC*</th><th>*AF*</th></tr><tr><td>1</td><td>2</td><td>$\sqrt{20}$</td></tr><tr><td>1</td><td>3</td><td>$\sqrt{15}$</td></tr><tr><td>2</td><td>3</td><td>$\sqrt{12}$</td></tr></table> eg none of the values of AF are integers and all possibilities have been covered.	M1(AO2.1) A1(AO1.1) E1(AO2.4) [3]	attempt at proof by exhaustion all cases covered ignore cases where AF < BC supporting comment may be brief but must refer to values of *AF*	

			Total	3		
5			Suppose that for the given perimeter, $4L$, there is a rectangle which is larger in area than the square. There is a square which has the same area as this rectangle but a smaller perimeter so its side is less than L. The square with side L has perimeter $4L$ and an area larger than the given rectangle. This is a contradiction so the square must have the largest area of all rectangles with given perimeter.	M1 (AO 2.5) A1 (AO 3.1a) A1 (AO 2.2a) [3]	Setting up a statement for contradiction. Use of statement in line 31–32 Completion to correct conclusion, including contradiction **Examiner's Comments** Proof by contradiction was challenging for the majority of candidates. Many offered no solution and some tried a proof by deduction. Those candidates that were successfully were able to make progress from a clear, suitable initial statement for contradiction.	
			Total	3		
6			$x^2+x+2=\left(x+\frac{1}{2}\right)^2+1\frac{3}{4}$ Minimum value is $1\frac{3}{4}$ hence always greater than 1 **Alternative solution 1** $y=x^2+x+2\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=2x+1$ Minimum occurs when $x=-\frac{1}{2}$ Minimum value is $1\frac{3}{4}$ hence always greater than 1 **Alternative solution 2** Discriminant of x^2+x+1 is $1-4$ $=-3$ Discriminant negative so x^2+x+1 is never zero,	M1 (AO 3.1a) A1 (AO 1.1) E1 (AO 2.4) M1 A1 E1 M1 A1	Attempt to complete square Correct completed square form Conclusion clearly explained Conclusion	Or equivalent steps, following initial rearrangement as $x^2+x+1>0$ Or equivalent steps, following initial rearrangement as $x^2+x+1>0$

			when $x = 0$, $x^2 + x + 1 = 1$, hence always positive.	E1 [3]	clearly explained Must be working on $x^2 + x + 1 > 0$ Complete argument		
			Total	3			
7			% increase in price $= \frac{100k}{P}$ $\text{PED} = \frac{100h}{Q} \div \frac{100k}{P} = \frac{hP}{kQ}$ $\frac{k}{h} = m$ so $\text{PED} = \frac{P}{mQ}$	B1 (AO 1.1a) M1 (AO 2.2a) E1 (AO 2.1) [3]	**AG** Correct working needed	Lose 1 mark max for sign errors in this question	
			Total	3			