Questions

Q1.

(i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

(Total for question = 5 marks)

Q2.

(a) Prove that for all positive values of x and y

$$\sqrt{xy} \leq \frac{x+y}{2}$$

(2)

(b) Prove by counter example that this is not true when x and y are both negative.

(1)

(Total for question = 3 marks)

Q3.

Given $n \in \mathbb{N}$, , prove that $n^3 + 2$ is not divisible by 8.

(Total for question = 4 marks)

Q4.

(a) Prove that for all positive values of *a* and *b*

$$\frac{4a}{b} + \frac{b}{a} \geqslant 4$$

(4)

(b) Prove, by counter example, that this is not true for all values of *a* and *b*.

(1)

(Total for question = 5 marks)

Q5.

A student is investigating the following statement about natural numbers.

"$n^3 - n$ is a multiple of 4"

(a) Prove, using algebra, that the statement is true for all odd numbers.

(4)

(b) Use a counterexample to show that the statement is not always true.

(1)

(Total for question = 5 marks)

Q6.

Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. (2)				
(ii) If $ax > b$ then $x > \frac{b}{a}$ (2)				
(iii) The difference between consecutive square numbers is odd. (2)				

(Total for question = 6 marks)

Q7.

Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(Total for question = 4 marks)

Q8.

Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

(Total for question = 4 marks)

Q9.

(i) Use proof by exhaustion to show that for $n \in \mathbb{N}, n \leqslant 4$

$$(n + 1)^3 > 3^n$$

(2)

(ii) Given that $m^3 + 5$ is odd, use proof by contradiction to show, using algebra, that m is even.

(4)

(Total for question = 6 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2\geqslant 0 \Rightarrow (x-4)^2+1\geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

Notes

(i) Method One: Completing the Square

M1: For an attempt to complete the square. Accept $(x-4)^2$ ...

A1: For $(x-4)^2+1$ with either $(x-4)^2\geqslant 0, (x-4)^2+1\geqslant 1$ or min at (4,1). Accept the inequality statements in words. Condone $(x-4)^2>0$ or a squared number is always positive for this mark.

A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and a conclusion

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$x^2-8x+17$
$=(x-4)^2+1\geqslant 1$ as $(x-4)^2\geqslant 0$ — scores M1 A1 A1
Hence $(x-4)^2+1>0$

$x^2-8x+17>0$
$(x-4)^2+1>0$ — scores M1 A1 A1
This is true because $(x-4)^2\geqslant 0$ and when you add 1 it is going to be positive

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$x^2-8x+17>0$
$(x-4)^2+1>0$ — scores M1 A1 A0
which is true because a squared number is positive — incorrect and incomplete

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$x^2-8x+17=(x-4)^2+1$ — scores M1 A1 A0
Minimum is (4,1) so $x^2-8x+17>0$ — correct but not explained

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$x^2-8x+17=(x-4)^2+1$ — scores M1 A1 A1
Minimum is (4,1) so as $1>0 \Rightarrow x^2-8x+17>0$ — correct and explained

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$x^2 - 8x + 17 > 0$

$(x-4)^2 + 1 > 0$

scores M1 A0 (no explanation) A0

Method Two: Use of a discriminant

M1: Attempts to find the discriminant $b^2 - 4ac$ with a correct *a*, *b* and *c* which may be within a quadratic formula. You may condone missing brackets.

A1: Correct value of $b^2 - 4ac = -4$ **and** states or shows curve is U shaped (or intercept is (0,17)) or equivalent such as +ve x^2 etc

A1: Explains that as $b^2 - 4ac < 0$, there are no roots, and curve is U shaped then $x^2 - 8x + 17 > 0$

Method Three: Differentiation

M1: Attempting to differentiate and finding the turning point. This would involve attempting to find $\frac{dy}{dx}$, then setting it equal to 0 and solving to find the *x* value and the *y* value.

A1: For differentiating $\frac{dy}{dx} = 2x - 8 \Rightarrow (4,1)$ **is the turning point**

A1: Shows that $(4,1)$ is the minimum point (second derivative or U shaped), hence $x^2 - 8x + 17 > 0$

Method 4: Sketch graph using calculator

M1: Attempting to sketch $y = x^2 - 8x + 17$, U shape with minimum in quadrant one

A1: As above with minimum at (4,1) marked

A1: Required to state that quadratics only have one turning point and as "1" is above the *x*-axis then $x^2 - 8x + 17 > 0$

(ii)

Numerical approach

Do not allow any marks if the student just mentions "positive" and "negative" numbers. Specific examples should be seen calculated if a numerical approach is chosen.

M1: Attempts a value (where it is not true) and shows/implies that it is not true for that value. For example, for -4 : $(-4+3)^2 > (-4)^2$ and indicates not true (states not true, ×)

or writing $(-4+3)^2 < (-4)^2$ is sufficient to imply that it is not true

A1: Shows/implies that it can be true for a value **AND** states sometimes true. For example for +4 : $(4+3)^2 > 4^2$ and indicates true ✓

or writing $(4+3)^2 > 4^2$ is sufficient to imply this is true following $(-4+3)^2 < (-4)^2$

condone incorrect statements following the above such as 'it is only true for positive numbers' as long as they state "sometimes true" and show both cases.

Algebraic approach

M1: Sets the problem up algebraically Eg. $(x+3)^2 > x^2 \Rightarrow x > k$ Any inequality is fine. You may condone one error for the method mark. Accept $(x+3)^2 > x^2 \Rightarrow 6x + 9 > 0$ oe

A1: States sometimes true **and** states/implies true for $x > -\frac{3}{2}$ or states/implies not true for $x \le -\frac{3}{2}$ In both cases you should expect to see the statement "sometimes true" to score the A1

Q2.

Question	Scheme	Marks	AOs
(a) Way 1	Since x and y are positive, their square roots are real and so $(\sqrt{x}-\sqrt{y})^2 \geq 0$ giving $x-2\sqrt{x}\sqrt{y}+y \geq 0$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x+y$ provided x and y are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ *	A1*	2.2a
		(2)	
Way 2 Longer method	Since $(x-y)^2 \geq 0$ for real values of x and y, $x^2-2xy+y^2 \geq 0$ and so $4xy \leq x^2+2xy+y^2$ i.e. $4xy \leq (x+y)^2$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x+y$ provided x and y are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ *	A1*	2.2a
		(2)	
(b)	Let $x=-3$ and $y=-5$ then LHS $=\sqrt{15}$ and RHS$=-4$ so as $\sqrt{15}>-4$ result does not apply	B1	2.4
		(1)	
		(3 marks)	

Notes

(a) M1 : Need two stages of the three stage argument involving the three stages, squaring, square rooting terms and rearranging.

A1*: Need all three stages making the correct deduction to achieve the printed result.

(b) B1 : Chooses two negative values and substitutes, then states conclusion

Q3.

Score as below so M0 A0 M1 A1 or M1 A0 M1 A1 are not possible

Generally the marks are awarded for
M1: Suitable approach to answer the question for n being even **OR** odd
A1: Acceptable proof for n being even **OR** odd
M1: Suitable approach to answer the question for n being even **AND** odd
A1: Acceptable proof for n being even **AND** odd **WITH** concluding statement.

There is no merit in a

- student taking values, or multiple values, of n and then drawing conclusions. So $n=5 \Rightarrow n^3+2=127$ which is not a multiple of 8 scores no marks.
- student using divided when they mean divisible. Eg. "Odd numbers cannot be divided by 8" is incorrect. We need to see either "odd numbers are not divisible by 8" or "odd numbers cannot be divided by 8 **exactly**"
- stating $\frac{n^3+2}{8}=\frac{1}{8}n^3+\frac{1}{4}$ which is not a whole number
- stating $\frac{(n+1)^3+2}{8}=\frac{1}{8}n^3+\frac{3}{8}n^2+\frac{3}{8}n+\frac{3}{8}$ which is not a whole number

There must be an attempt to generalise either logic or algebra.

Example of a logical approach

Logical approach	States that if n is odd, n^3 is odd	M1	2.1
	so n^3+2 is odd and therefore cannot be divisible by 8	A1	2.2a
	States that if n is even, n^3 is a multiple of 8	M1	2.1
	so n^3+2 cannot be a multiple of 8 So (Given $n \in \mathbb{N}$), n^3+2 is not divisible by 8	A1	2.2a
		(4)	
			4 marks

First M1: States the result of cubing an odd or an even number
First A1: Followed by the result of adding two and gives a valid reason why it is not divisible by 8.
So for odd numbers accept for example
"odd number + 2 is still odd and odd numbers are not divisible by 8"
"n^3+2 is odd and cannot be divided by 8 **exactly**"
and for even numbers accept
"a multiple of 8 add 2 is not a multiple of 8, so n^3+2 is not divisible by 8"
"if n^3 is a multiple of 8 then n^3+2 cannot be divisible by 8"
Second M1: States the result of cubing an odd and an even number
Second A1: Both valid reasons must be given followed by a concluding statement.

Example of algebraic approaches

Question	Scheme	Marks	AOs
Algebraic approach	(If n is even,) $n = 2k$ and $n^3 + 2 = (2k)^3 + 2 = 8k^3 + 2$	M1	2.1
	Eg.'This is 2 more than a multiple of 8, hence not divisible by 8' Or 'as $8k^3$ is divisible by 8 , $8k^3 + 2$ isn't'	A1	2.2a
	(If n is odd,) $n = 2k+1$ and $n^3 + 2 = (2k+1)^3 + 2$	M1	2.1
	$= \underline{\underline{8k^3 + 12k^2 + 6k + 3}}$ which is an even number add 3, therefore odd. Hence it is not divisible by 8 So (given $n \in \mathbb{N}$,) $n^3 + 2$ is not divisible by 8	A1	2.2a
		(4)	
Alt algebraic approach	(If n is even,) $n = 2k$ and $\frac{n^3+2}{8} = \frac{(2k)^3+2}{8} = \frac{8k^3+2}{8}$	M1	2.1
	$= k^3 + \frac{1}{4}$ oe which is not a whole number and hence not divisible by 8	A1	2.2a
	(If n is odd,) $n = 2k+1$ and $\frac{n^3+2}{8} = \frac{(2k+1)^3+2}{8}$	M1	2.1
	$= \frac{8k^3+12k^2+6k+3}{8}$ ** The numerator is odd as $\underline{\underline{8k^3 + 12k^2 + 6k + 3}}$ is an even number +3 hence not divisible by 8 So (Given $n \in \mathbb{N}$,) $n^3 + 2$ is not divisible by 8	A1	2.2a
		(4)	

Notes
Correct expressions are required for the M's. There is no need to state "**If *n* is even,**" $n = 2k$ and "**If *n* is odd,** $n = 2k+1$" for the two M's as the expressions encompass all numbers. However the concluding statement must attempt to show that it has been proven for all $n \in \mathbb{N}$ Some students will use $2k-1$ for odd numbers   There is no requirement to change the variable. They may use $2n$ and $2n \pm 1$ Reasons must be correct. Don't accept $8k^3 + 2$ cannot be divided by 8 for example. (It can!)   Also **" $= \frac{8k^3+12k^2+6k+3}{8} = k^3 + \frac{3}{2}k^2 + \frac{3}{4}k + \frac{3}{8}$ which is not whole number" is too vague so A0

Q4.

Question	Scheme	Marks	AOs
(a)	States $(2a-b)^2 .. 0$	M1	2.1
	$4a^2+b^2 .. 4ab$	A1	1.1b
	(As $a>0, b>0$) $\frac{4a^2}{ab}+\frac{b^2}{ab} .. \frac{4ab}{ab}$	M1	2.2a
	Hence $\frac{4a}{b}+\frac{b}{a} .. 4$ * CSO	A1*	1.1b
		(4)	
(b)	$a=5, b=-1 \Rightarrow \frac{4a}{b}+\frac{b}{a}=-20-\frac{1}{5}$ which is less than 4	B1	2.4
		(1)	
			(5 marks)

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 .. 0$

A1: Reaches $4a^2+b^2 \ldots 4ab$

M1: Divides each term by $ab \Rightarrow \frac{4a^2}{ab}+\frac{b^2}{ab} .. \frac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a>0$ and $b>0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

Proof by contradiction: Scores all marks

M1: Assume that there exists an $a, b > 0$ such that $\frac{4a}{b} + \frac{b}{a} < 4$

A1: $4a^2 + b^2 < 4ab \Rightarrow 4a^2 + b^2 - 4ab < 0$

M1: $(2a - b)^2 < 0$

A1*: States that this is not true, hence we have a contradiction so $\frac{4a}{b} + \frac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a > 0$ and $b > 0$

Attempt starting with the left-hand side

M1: $(\text{lhs} =) \frac{4a}{b} + \frac{b}{a} - 4 = \frac{4a^2 + b^2 - 4ab}{ab}$

A1: $= \frac{(2a - b)^2}{ab}$

M1: $= \frac{(2a - b)^2}{ab} \ldots 0$

A1*: Hence $\frac{4a}{b} + \frac{b}{a} - 4 \ldots 0 \Rightarrow \frac{4a}{b} + \frac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- ab is positive as $a > 0$ and $b > 0$

Attempt using given result: For 3 out of 4

$\frac{4a}{b} + \frac{b}{a} \ldots 4$ M1 $\Rightarrow 4a^2 + b^2 \ldots 4ab \Rightarrow 4a^2 + b^2 - 4ab \ldots 0$

A1 $\Rightarrow (2a - b)^2 \ldots 0$ oe

M1 gives both reasons why this is true

- "square numbers are greater than or equal to 0"
- "multiplying by ab does not change the sign of the inequality because a and b are positive"

Q5.

Question	Scheme	Marks	AOs
(a)	Selects a correct strategy. E.g uses an odd number is $2k \pm 1$	B1	3.1a
	Attempts to simplify $(2k \pm 1)^3 - (2k \pm 1) = \ldots$	M1	2.1
	..........and factorise $8k^3 \pm 12k^2 \pm 4k = 4k\left(2k^2 \pm 3k \pm 1\right) =$	dM1	1.1b
	Correct work with statement $4 \times ..$ is a multiple of 4	A1	2.4
		(4)	
(b)	Any counter example with correct statement. Eg. $2^3 - 2 = 6$ which is not a multiple of 4	B1	2.4
		(1)	
		(5 marks)	
Alt (a)	Selects a correct strategy. Factorises $k^3 - k = k(k-1)(k+1)$	B1	3.1a
	States that if k is odd then both $k-1$ and $k+1$ are even	M1	2.1
	States that $k-1$ multiplied by $k+1$ is therefore a multiple of 4	dM1	1.1b
	Concludes that $k^3 - k$ is a multiple of 4 as it is odd $\times$ multiple of 4	A1	2.4
		(4)	

Notes:

(a)

Note: May be in any variable (condone use of n)

B1: Selects a correct strategy. E.g uses an odd number is $2k \pm 1$

M1: Attempts $(2k \pm 1)^3 - (2k \pm 1) = \ldots$ Condone errors in multiplying out the brackets and invisible brackets for this mark. Either the coefficient of the k term or the constant of $(2k \pm 1)^3$ must have changed from attempting to simplify.

dM1: Attempts to take a factor of 4 or $4k$ from their cubic

A1: Correct work with statement $4 \times ..$ is a multiple of 4

(b)

B1: Any counter example with correct statement.

Q6.

Question	Scheme	Marks	AOs
(i)	$x^2-6x+10=(x-3)^2+1$	M1	2.1
	Deduces "always true" as $(x-3)^2\geqslant 0 \Rightarrow (x-3)^2+1\geqslant 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a>0$ then $ax>b\Rightarrow x>\frac{b}{a}$ if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $=(n+1)^2-n^2=2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1$ = (even +1) = odd	A1	2.2a
		(2)	
			(6 marks)

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y=x^2-6x+10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$ or simply $-3x>6\Rightarrow x<-2$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.
For example by attempting $(n+1)^2-n^2=2n+1$ or $m^2-n^2=(m-n)(m+n)$ with $m=n+1$

A1: States always true with reason and proof
Accept a proof written in words. For example
If integers are consecutive, one is odd and one is even
When squared $\text{odd}\times\text{odd}=\text{odd}$ and $\text{even}\times\text{even}=\text{even}$
The difference between odd and even is always odd, hence always true
Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

Q7.

Question	Scheme	Marks	AOs
	Sets up the contradiction and factorises: There are positive integers p and q such that $(2p+q)(2p-q)=25$	M1	2.1
	If true then $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ **Award for deducing either of the above statements**	M1	2.2a
	Solutions are $p=6.5, q=12$ or $p=2.5, q=0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2-q^2=25$	A1	2.1
		(4)	
			(4 marks)
Notes:			

M1: For the key step in setting up the contradiction and factorising

M1: For deducing that for p and q to be integers then either $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ must be true.

Award for deducing either of the above statements.

You can ignore any reference to $\begin{matrix}2p+q=1\\2p-q=25\end{matrix}$ as this could not occur for positive p and q.

A1: For correctly solving one of the given statements,

For $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ candidates only really need to proceed as far as $p=6.5$ to show the contradiction.

For $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates only really need to find either p or q to show the contradiction.

Alt for $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates could state that $2p+q\neq 2p-q$ if p,q are positive integers.

A1: For a complete and rigorous argument with both possibilities and a correct conclusion.

Question	Scheme	Marks	AOs
Alt 1	Sets up the contradiction, attempts to make q^2 or $4p^2$ the subject and states that either $4p^2$ is even(*) , or that q^2 (or q) is odd (**) Either There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow q^2 = 4p^2 - 25$ with * or ** Or There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow 4p^2 = q^2 + 25$ with * or **	M1	2.1
	Sets $q = 2n \pm 1$ and expands $(2n \pm 1)^2 = 4p^2 - 25$	M1	2.2a
	Proceeds to an expression such as $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n + 6) + 2$ $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n) + \frac{13}{2}$ $p^2 = n^2 + n + \frac{13}{2}$	A1	1.1b
	States This is a contradiction as $4p^2$ must be a multiple of 4 Or p^2 must be an integer And concludes there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	

Alt 2

An approach using odd and even numbers is unlikely to score marks.

To make this consistent with the Alt method, score

M1: Set up the contradiction and start to consider one of the cases below where q is odd, $m \neq n$.

Solutions using the same variable will score no marks.

M1: Set up the contradiction and start to consider BOTH cases below where q is odd, $m \neq n$.

No requirement for evens

A1: Correct work and deduction for one of the two scenarios where q is odd

A1: Correct work and deductions for both scenarios where q is odd with a final conclusion

Options	Example of Calculation	Deduction
p (even) q (odd)	$4p^2 - q^2 = 4 \times (2m)^2 - (2n+1)^2 = 16m^2 - 4n^2 - 4n - 1$	One less than a multiple of 4 so cannot equal 25
p (odd) q (odd)	$4p^2 - q^2 = 4 \times (2m+1)^2 - (2n+1)^2 = 16m^2 + 16m - 4n^2 - 4n + 3$	Three more than a multiple of 4 so cannot equal 25

Q8.

Question	Scheme	Marks	AOs
	NB any natural number can be expressed in the form: $3k,\ 3k+1,\ 3k+2$ or equivalent e.g. $3k-1,\ 3k,\ 3k+1$		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(3k)^2 = 9k^2\left(=3\times 3k^2\right)$ is a multiple of 3	A1 **M1 on EPEN**	1.1b
	$(3k+1)^2 = 9k^2+6k+1 = 3\times\left(3k^2+2k\right)+1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2+12k+4 = 3\times\left(3k^2+4k+1\right)+1$ $\left(\text{or } (3k-1)^2 = 9k^2-6k+1 = 3\times\left(3k^2-2k\right)+1\right)$ is one more than a multiple of 3		
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k,\ 3k+1,\ 3k+2$ or e.g. $3k-1,\ 3k,\ 3k+1$	M1 **A1 on EPEN**	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
			(4 marks)

Notes:

M1: Makes the key step of attempting to write the natural numbers in any 2 of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square these expressions.

A1(M1 on EPEN): Successfully shows for 2 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 <u>using algebra</u>. This must be made explicit e.g. reaches $3\times\left(3k^2+2k\right)+1$ and makes a statement that this is one more than a multiple of 3 but also allow other rigorous arguments that reason why $9k^2+6k+1$ is one more than a multiple of 3 e.g. "$9k^2$ is a multiple of 3 and $6k$ is a multiple of 3 so $9k^2+6k+1$ is one more than a multiple of 3"

M1(A1 on EPEN): Recognises that all natural numbers can be written in one of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square in all 3 cases.

A1: Successfully shows for all 3 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra and makes a conclusion

Q9.

Question	Scheme	Marks	AOs
(i)	$n=1,\ 2^3=8,\ 3^1=3,\ (8>3)$ $n=2,\ 3^3=27,\ 3^2=9,\ (27>9)$ $n=3,\ 4^3=64,\ 3^3=27,\ (64>27)$ $n=4,\ 5^3=125,\ 3^4=81,\ (125>81)$	M1	2.1
	So if $n \leqslant 4, n \in \mathbb{N}$ then $(n+1)^3 > 3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m=(2p\pm1)$ and attempt $m^3+5=(2p\pm1)^3+5=\ldots$	M1	2.1
	$=8p^3+12p^2+6p+6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion • reason for $8p^3+12p^2+6p+6$ being even • acceptable statement such as "this is a contradiction so if m^3+5 is odd then m must be even"	A1	2.4
		(4)	
			(6 marks)

Notes

(i)

M1: A full and rigorous argument that uses all of n = 1, 2, 3 and 4 in an attempt to prove the given result. Award for attempts at both $(n+1)^3$ and 3^n for **ALL** values with at least 5 of the 8 values correct.

There is no requirement to compare their sizes, for example state that $27>9$

Extra values, say $n=0$, may be ignored

A1: Completes the proof with no errors and an appropriate/allowable conclusion.

This requires

- all the values for n =1, 2, 3 and 4 correct. Ignore other values
- all pairs compared correctly
- a minimal conclusion. Accept ✓ or hence proven for example

(ii)

M1: Begins the proof by negating the statement. See scheme

This cannot be scored if the candidate attempts m both odd and even

M1: For the key step in setting $m=2p\pm1$ and attempting to expand $(2p\pm1)^3+5$

Award for a 4 term cubic expression.

A1: Correctly reaches $(2p+1)^3+5=8p^3+12p^2+6p+6$ and states even.

Alternatively reaches $(2p-1)^3+5=8p^3-12p^2+6p+4$ and states even.

A1: A full and complete argument that completes the contradiction proof. See scheme.

(1) **A reason** why the expression $8p^3+12p^2+6p+6$ or $8p^3-12p^2+6p+4$ is even

Acceptable reasons are

- all terms are even
- sight of a factorised expression E.g. $8p^3-12p^2+6p+4=2(4p^3-6p^2+3p+2)$

(2) Acceptable concluding statement

Acceptable concluding statements are

- "this is a contradiction, so if m^3+5 is odd then m is even"
- "this is contradiction, so proven."
- "So if m^3+5 is odd them m is even"