NUMERICAL METHODS — Answers

1 **a** $\frac{dy}{dx} = e^x + 2x$

b at A, $x = 0$ $\therefore$ $y = -3$, grad = 1

$\therefore$ $y = x - 3$

c SP: $e^x + 2x = 0$

let $f(x) = e^x + 2x$

$f(-0.4) = -0.130$

$f(-0.3) = 0.141$

sign change, $f(x)$ continuous $\therefore$ root

$\therefore$ x-coord of B in interval $[-0.4, -0.3]$

d $x_1 = -0.34694$

$x_2 = -0.35126$

$x_3 = -0.35169$

$x_4 = -0.35173$

$\therefore$ x-coord of $B = -0.352$ (3dp)

2 **a** $f(0) = 0.279$

$f(5) = -4.10$

$f(1) = 0.266$

$f(3) = -2.44$

$f(2) = -0.853$

$\therefore$ $k = 1$

b $x_0 = 1$

$x_1 = 1.2684$

$x_2 = 1.3106$

$x_3 = 1.3106$

3 **a** area of segment $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}r^2(\theta - \sin\theta)$

$\therefore$ $\frac{1}{2}r^2\sin\theta = 4 \times \frac{1}{2}r^2(\theta - \sin\theta)$

$\sin\theta = 4(\theta - \sin\theta)$

$\sin\theta = 4\theta - 4\sin\theta$

$4\theta - 5\sin\theta = 0$

b $\theta_1 = 1.11401$

$\theta_2 = 1.12184$

$\theta_3 = 1.12613$

$\theta_4 = 1.12844$

$\theta_5 = 1.12968$

$\therefore$ $\theta = 1.13$ (2dp)

4 **a** $e^{x^2} - x - 3 = 0$

$e^{x^2} = x + 3$

$x^2 = \ln(x + 3)$

$x = \sqrt{\ln(x+3)}$ $\therefore$ $a = 1$, $b = 3$

b e.g. $x_0 = 1.5$

$x_1 = 1.226408$

$x_2 = 1.200563$

$x_3 = 1.198006$

$x_4 = 1.197752$

$x_5 = 1.197727$

$\therefore$ solution = 1.198 (3dp)

5 **a** $y = x^2 - 9$

swap $x = y^2 - 9$

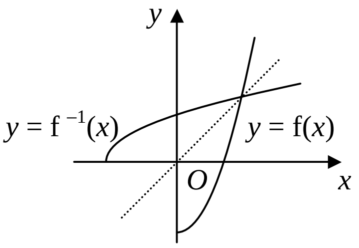
$y = \pm\sqrt{x+9}$

(domain ⇒ +)

$f^{-1}(x) = \sqrt{x+9}$, $x \in \mathbb{R}$, $x \geq -9$

range: $f^{-1}(x) \geq 0$

b



c let $h(x) = f^{-1}(x) + g(x) = \sqrt{x+9} + x^3$

$h(-2) = -5.35$

$h(-1) = 1.83$

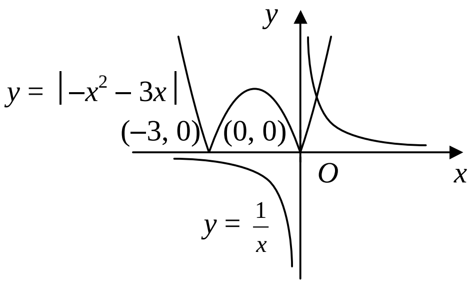
sign change, $h(x)$ continuous ∴ root

d $x_1 = -1.41421$, $x_2 = -1.40174$,

$x_3 = -1.40212$, $x_4 = -1.40211$

∴ root = −1.402 (3dp)

6 **a**



b $-(-x^2 - 3x) = \frac{1}{x}$

$x^2 + 3x = \frac{1}{x}$

$x^3 + 3x^2 = 1$

$x^3 + 3x^2 - 1 = 0$

c $x_1 = 0.57735$

$x_2 = 0.52871$

$x_3 = 0.53234$

$x_4 = 0.53207$

∴ x-coord of P = 0.532 (3dp)

7 **a** at A, $x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x = 0$

let $f(x) = x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x$

$f(4) = -2$, $f(5) = 14.2$

sign change, $f(x)$ continuous ∴ root

∴ $4 < \alpha < 5$

b $\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7$

at B, $\frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7 = 0$

let $g(x) = \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7$

$g(2) = -0.990$, $g(3) = 5.12$

sign change, $g(x)$ continuous ∴ root

∴ $2 < \beta < 3$

c $\frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7 = 0$

$5x^2 - 3 - 14x^{\frac{1}{2}} = 0$

$x^2 = 0.6 + 2.8x^{\frac{1}{2}}$

$x > 0$ ∴ $x = \beta$ is a soln to $x = \sqrt{0.6 + 2.8x^{\frac{1}{2}}}$

d $x_1 = 2.158144$

$x_2 = 2.171031$

$x_3 = 2.173853$

$x_4 = 2.174470$

$x_5 = 2.174604$

∴ $\beta = 2.175$ (4sf)

8 **a** $\frac{dy}{dx} = 3 - \frac{1}{x}$

grad = 2

∴ grad of normal = $-\frac{1}{2}$

∴ $y - 3 = -\frac{1}{2}(x - 1)$

[$y = \frac{7}{2} - \frac{1}{2}x$]

b $3x - \ln x = \frac{7}{2} - \frac{1}{2}x$

$6x - 2\ln x = 7 - x$

$2\ln x - 7x + 7 = 0$

c $2\ln x = 7x - 7$

$\ln x = \frac{7}{2}(x - 1)$

$x = e^{\frac{7}{2}(x-1)}$ ∴ $k = \frac{7}{2}$

d $x_1 = 0.173774$

$x_2 = 0.055477$

$x_3 = 0.036669$

$x_4 = 0.034333$

$x_5 = 0.034053$

∴ x-coord of Q = 0.034 (3dp)

e let $f(x) = 2\ln x - 7x + 7$

$f(0.0335) = -0.027$

$f(0.0345) = 0.025$

sign change, $f(x)$ continuous ∴ root