NUMERICAL METHODS — *Answers*

1 **a** $9 + 4x - 2x^3 = 0$

$x^3 = 2x + 4.5$

$x = \sqrt[3]{2x + 4.5}$

$\therefore x_{n+1} = \sqrt[3]{2x_n + 4.5}$

$x_1 = 2.040828$

$x_2 = 2.047342$

$x_3 = 2.048377 = 2.0484$ (4dp)

b $e^x - 8x + 5 = 0$

$e^x = 8x - 5$

$x = \ln(8x - 5)$

$\therefore x_{n+1} = \ln(8x_n - 5)$

$x_1 = 2.944439$

$x_2 = 2.920767$

$x_3 = 2.910508 = 2.9105$ (4dp)

c $\tan x - 5x + 13 = 0$

$\tan x = 5x - 13$

$x = \arctan(5x - 13)$

$\therefore x_{n+1} = \arctan(5x_n - 13)$

$x_1 = -1.518213$

$x_2 = -1.522270$

$x_3 = -1.522317 = -1.5223$ (4dp)

d $\ln x + \sqrt{x} + 1.4 = 0$

$\ln x = -(\sqrt{x} + 1.4)$

$x = e^{-(\sqrt{x}+1.4)}$

$\therefore x_{n+1} = e^{-(\sqrt{x_n}+1.4)}$

$x_1 = 0.165299$

$x_2 = 0.164216$

$x_3 = 0.164436 = 0.1644$ (4dp)

2 **a** $e^{2x-1} - 6x = 0$

$e^{2x-1} = 6x$

$2x - 1 = \ln 6x$

$x = \frac{1}{2}(\ln 6x + 1)$

$\therefore x_{n+1} = \frac{1}{2}(\ln 6x_n + 1),\ a = \frac{1}{2},\ b = 6$

$x_1 = 1.661194$

$x_2 = 1.649648$

$x_3 = 1.646161 = 1.646$ (3dp)

b $\frac{2}{x} + \cos x - 3 = 0$

$\frac{2}{x} = 3 - \cos x$

$2 = x(3 - \cos x)$

$x = \frac{2}{3 - \cos x}$

$\therefore x_{n+1} = \frac{2}{3 - \cos x_n},\ a = 2,\ b = 3$

$x_1 = 0.868322$

$x_2 = 0.849657$

$x_3 = 0.854789 = 0.855$ (3dp)

c $2x^3 - 6x - 11 = 0$

$2x^3 = 6x + 11$

$x^2 = 3 + \frac{11}{2x}$

$x = \pm\sqrt{3 + \frac{11}{2x}}$

$\therefore x_{n+1} = \sqrt{3 + \frac{5.5}{x_n}},\ a = 3,\ b = 5.5$

$x_1 = 2.397916$

$x_2 = 2.300795$

$x_3 = 2.321740 = 2.322$ (3dp)

d $15\ln(x + 3) - 4x = 0$

$\ln(x + 3) = \frac{4}{15}x$

$x + 3 = e^{\frac{4}{15}x}$

$x = e^{\frac{4}{15}x} - 3$

$\therefore x_{n+1} = e^{\frac{4}{15}x_n} - 3,\ a = \frac{4}{15},\ b = -3$

$x_1 = -2.486583$

$x_2 = -2.484743$

$x_3 = -2.484490 = -2.484$ (3dp)

3 **a** $x_1 = 0.428135$
$x_2 = 0.433865$
$x_3 = 0.431107$
$x_4 = 0.432437$
$x_5 = 0.431796$
$\therefore$ root = 0.432 (3dp)
f(0.4315) = –0.00465
f(0.4325) = 0.00457
sign change, f(x) continuous $\therefore$ root

b $x_1 = 0.474342$
$x_2 = 0.470474$
$x_3 = 0.469923$
$\therefore$ root = 0.47 (2sf)
f(0.465) = –0.00428
f(0.475) = 0.00463
sign change, f(x) continuous $\therefore$ root

c $x_1 = 5.892685$
$x_2 = 5.859202$
$x_3 = 5.850013$
$x_4 = 5.847607$
$x_5 = 5.846985$
$x_6 = 5.846825$
$\therefore$ root = 5.85 (3sf)
f(5.845) = 0.00658
f(5.855) = –0.0305
sign change, f(x) continuous $\therefore$ root

d $x_1 = 3.731246$
$x_2 = 3.724839$
$x_3 = 3.726145$
$x_4 = 3.725879$
$\therefore$ root = 3.726 (3dp)
f(3.7255) = 0.000672
f(3.7265) = –0.000912
sign change, f(x) continuous $\therefore$ root

4 **a** $x_1 = -3.192595$
$x_2 = -3.188214$
$x_3 = -3.185620$
$x_4 = -3.184084$
$x_5 = -3.183174$
$x_6 = -3.182635$
$\therefore$ root = –3.18 (2dp)

b $x^5 - 10x^3 + 4 = 0$
$4 = 10x^3 - x^5 = x^3(10 - x^2)$
$x^3 = \dfrac{4}{10 - x^2}$
$x = \sqrt[3]{\dfrac{4}{10 - x^2}}$ $\therefore$ $a = 4$, $b = 10$

c $x_1 = 0.763143$
$x_2 = 0.751692$
$x_3 = 0.751231$
$x_4 = 0.751212$
$\therefore$ root = 0.751 (3dp)

5 **a** $\arcsin 2x - 0.5x - 0.7 = 0$
$\arcsin 2x = 0.5x + 0.7$
$2x = \sin(0.5x + 0.7)$
$x = 0.5 \sin(0.5x + 0.7)$ $\therefore$ $a = 0.5$, $b = 0.5$, $c = 0.7$

b $x_1 = 0.391663$
$x_2 = 0.390365$
$x_3 = 0.390162$
$x_4 = 0.390130$
$\therefore$ solution = 0.390 (3dp)