

NUMERICAL METHODS

- 1** Show in each case that there is a root of the equation $f(x) = 0$ in the given interval.
- a** $f(x) = x^3 + 3x - 7$ (1, 2) **b** $f(x) = 5 \cos x - 3x$ (0.5, 1)
c $f(x) = 2e^x + x + 5$ (-6, -5) **d** $f(x) = x^4 - 5x^2 + 1$ (2.1, 2.2)
e $f(x) = \ln(4x - 1) + x^2$ (0.4, 0.5) **f** $f(x) = e^{-x} - 9 \cos 4x$ (10, 11)
- 2** Given that $|N| \leq 5$, find in each case the integer N such that there is a root of the equation $f(x) = 0$ in the interval $(N, N + 1)$.
- a** $f(x) = x^3 - 3\sqrt{x} - 4$ **b** $f(x) = x \ln x - \frac{12}{x}$ **c** $f(x) = 2x^5 + 4x + 15$
d $f(x) = e^{x-1} + 4x - 2$ **e** $f(x) = e^x - 3 \sin x$ **f** $f(x) = \tan(0.1x) + x - 6$
- 3** Show in each case that there is a root of the given equation in the given interval.
- a** $x^3 = 12 - \frac{x}{4}$ [2, 3] **b** $12e^x = 9 - 4x$ [-1, 0]
c $10 \ln 3x = 5 - 7x^2$ [0.47, 0.48] **d** $\sin 4x = 7e^x$ [-6.5, -6]
e $4^x = 3x + 10$ [-4, -3] **f** $\tan(\frac{1}{2}x) = 2x - 1$ [2.6, 2.7]
- 4** In each case there is a root of the equation $f(x) = 0$ in the given interval. Find the integer, a , such that this root lies in the interval $(\frac{a}{10}, \frac{a+1}{10})$.
- a** $f(x) = x^4 + \frac{3}{x} - 5$ (1, 2) **b** $f(x) = x - \ln(6 + x^2)$ (2, 3)
c $f(x) = 5x^3 - 3x^2 + 11$ (-2, -1) **d** $f(x) = \frac{8}{x} - \cos x$ (11, 12)
e $f(x) = \operatorname{cosec} x + \sqrt{x}$ (5, 6) **f** $f(x) = x^2 - 7e^{2x+5}$ (-3, -2)
- 5**
- a** On the same set of axes, sketch the graphs of $y = x^3$ and $y = 4 - x$.
b Hence, show that the equation $x^3 + x - 4 = 0$ has exactly one real root.
c Show that this root lies in the interval (1, 1.5).
- 6** $f: x \rightarrow x \ln x - 1, x \in \mathbb{R}, x > 0$.
- a** On the same set of axes, sketch the curves $y = \ln x$ and $y = \frac{1}{x}$.
b Hence show that the equation $f(x) = 0$ has exactly one real root.
The real root of $f(x) = 0$ is α .
c Find the integer n such that $n < \alpha < n + 1$.
- 7**
- a** On the same set of axes, sketch the curves $y = e^x$ and $y = 5 - x^2$.
b Hence show that the equation $e^x + x^2 - 5 = 0$ has exactly one negative and one positive real root.
c Show that the negative root lies in the interval (-3, -2).
The positive root, α , is such that $\frac{n}{10} < \alpha < \frac{n+1}{10}$, where n is an integer.
d Find the value of n .