1.

Fig. 3 shows the curve $y = x^3 + \sqrt{(\sin x)}$ for $0 \leqslant x \leqslant \frac{\pi}{4}$.

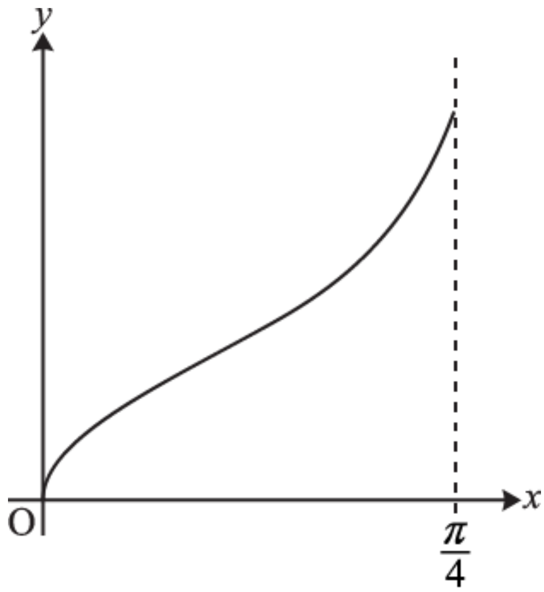

Fig. 3

i. Use the trapezium rule with 4 strips to estimate the area of the region bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, giving your answer to 3 decimal places.

[4]

ii. Suppose the number of strips in the trapezium rule is increased. Without doing further calculations, state, with a reason, whether the area estimate increases, decreases, or it is not possible to say.

[1]

2. Fig. 6 shows a partially completed spreadsheet. This spreadsheet uses the trapezium rule with four strips to estimate

$$\int_0^{\frac{1}{2}\pi} \sqrt{1+\sin x}\ \mathrm{d}x.$$

	A	B	C	D	E
1		x	$\sin x$	y	
2	0	0.0000	0.0000	1.0000	0.5000
3	0.125	0.3927	0.3827	1.1759	1.1759
4	0.25	0.7854	0.7071	1.3066	1.3066
5	0.375	1.1781	0.9239	1.3870	1.3870
6	0.5	1.5708	1.0000	1.4142	0.7071
7					5.0766
8					

Fig. 6

(a) Show how the value in cell B3 is calculated. [1]

(b) Show how the values in cells D2 to D6 are used to calculate the value in cell E7. [1]

(c) Complete the calculation to estimate $\int_0^{\frac{1}{2}\pi} \sqrt{1+\sin x}\,\mathrm{d}x$, giving the answer to 3 significant figures. [2]

3. Fig. 3 shows the curve $y=\sqrt{1+\mathrm{e}^{2x}}$.

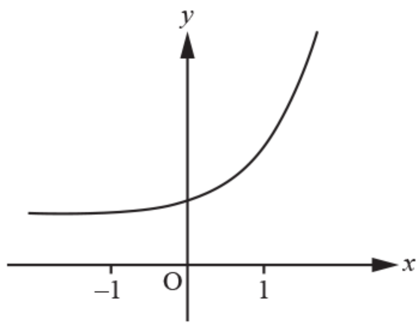

Fig. 3

The value of $\int_{-1}^{1} \sqrt{1+\mathrm{e}^{2x}}\,\mathrm{d}x$ is to be estimated using the trapezium rule. T_2 and T_4 are the estimates obtained from the trapezium rule using 2 strips and 4 strips respectively.

(i) Explain whether T_4 is greater or less than T_2. [2]

(ii) Evaluate T_4, giving your answer to 3 significant figures. [4]

4. The function f (x) is defined by $f(x) = \sqrt[3]{27 - 8x^3}$. Jenny uses her scientific calculator to create a table of values for f (x) and f ’(x).

x	f(x)	f'(x)
0	3	0
0.25	2.9954	–0.056
0.5	2.9625	–0.228
0.75	2.8694	–0.547
1	2.6684	–1.124
1.25	2.2490	–1.977
1.5	0	ERROR

(a) Use calculus to find an expression for f'(x) and hence explain why the calculator gives an error for f'(1.5). [3]

(b) Find the first three terms of the binomial expansion of f (x). [3]

(c) Jenny integrates the first three terms of the binomial expansion of f (x) to estimate the value of $\int_0^1 \sqrt[3]{27 - 8x^3}\,dx$. Explain why Jenny’s method is valid in this case. (You do not need to evaluate Jenny’s approximation.) [2]

(d) Use the trapezium rule with 4 strips to obtain an estimate for $\int_0^1 \sqrt[3]{27 - 8x^3}\,dx$. [3]

The calculator gives 2.921 174 38 for $\int_0^1 \sqrt[3]{27 - 8x^3}\,dx$. The graph of y = f ($x$) is shown in Fig. 13.

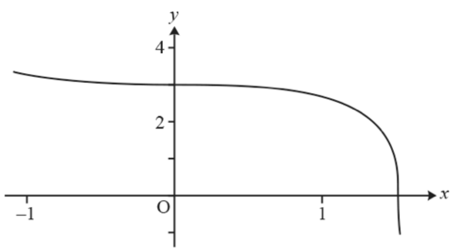

Fig. 13

(e) Explain why the trapezium rule gives an underestimate. [1]

5. Fig. 5.1 shows the curve $y = e^{1-x^2}$. Fig. 5.2 shows a spreadsheet used to calculate an estimate of $\int_0^2 e^{1-x^2}\,dx$ using the trapezium rule with four strips.

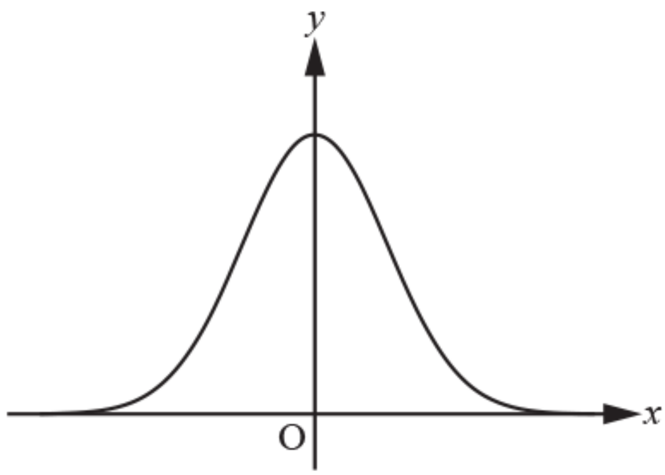

Fig. 5.1

	A	B	C
1	x		y
2	0	1	2.718282
3	0.5	0.75	2.117
4	1	0	1
5	1.5	-1.25	0.286505
6	2	-3	0.049787
7			

Fig. 5.2

(a) Show how the value in cell B3 is calculated. [1]

(b) Complete the calculation to estimate $\int_0^2 e^{1-x^2}\,dx$, giving the answer correct to 3 significant figures. [2]

(c) Show that the only stationary point on the curve is at (0, e). [2]

6.

Bob wishes to find an estimate for $\int_0^2 f(x)\,dx$, where $f(x) = \sqrt{x^{\frac{3}{2}} + 3}$, using the trapezium rule with 4 strips.

Fig. 6 is a screenshot of a spreadsheet Bob created to help him. In rows 2 to 6, the values in columns B and C have been multiplied to give the value in column D. The value in D7 is the sum of the values from D2 to D6.

	A	B	C	D
1	x	f(x)	multiplier	multiple of f(x)
2	0	1.732051	1	1.7321
3	0.5	1.831271	2	3.6625
4	1	2	2	4
5	1.5	2.199345	2	4.3987
6	2	2.414214	1	2.4142
7				16.2075

Fig. 6

(a) Calculate the estimate for $\int_0^2 \sqrt{x^{\frac{3}{2}} + 3}\,dx$ that Bob should obtain by using the trapezium rule with 4 strips. [2]

(b) You are given that the graph of $y = f(x)$ is concave upwards for $0 \le x \le 2$. Explain what you can deduce about the estimate for the integral obtained in part (a). [1]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1		i	<table><tr><td>x</td><td>0</td><td>0.1963</td><td>0.3927</td><td>0.5890</td><td>0.7854</td></tr><tr><td>y</td><td>0</td><td>0.4493</td><td>0.6792</td><td>0.9498</td><td>1.3254</td></tr></table>	B2, 1,0	For values 0.4493,0.6792,0.9498 (**4dp** or better soi) [accept truncated to 4 figs after dec point] [cannot assume values of form $(\pi/16)^3 + \sqrt{(\sin \pi/16)}$ are correct unless followed by correct total at some later stage as some will be in degree mode]
		i	$A = (\pi/32)\ [(0 + 1.3254) + 2(0.4493 + 0.6792 + 0.9498)]$	M1	Use of the trapezium rule. Trapezium rule formula for **4 strips** must be seen, with or without substitution seen. **Correct *h* must be soi.**
		i	= 0.538	A1	[accept separate trapezia added] 0.538 **www 3dp only** (NB using 1.325 is ww) SC B0 0.538 without any working as no indication of strips or method used SC B1 0.538 with some indication of 4 strips but no values seen Correct values followed by 0.538 scores B2 B0 Correct values followed by correct formula for 4 strips, with or without substitution seen, then A = 0.538 scores 4/4. Correct formula for 4 strips and values of form $((\pi/16)^3 + \sqrt{(\sin\pi/16)}$… followed by correct answer scores 4/4 (or ¾ with wrong dp) **NB Values given in the table to only 3dp give apparently the correct answer, but scores B0,M1A0 ww** **Examiner's Comments** Many errors were seen here. In a number of cases the candidates were in degree mode. For others *h* was given incorrectly. Many others used the wrong formula and some substituted *x* values in the formula or omitted 0 from the formula. However, probably the most common error was giving the *y* values to 3dp and then using these to give a final answer correct to 3dp.
		ii	Not possible to say, eg some trapezia are above and some below curve oe.	B1	Need a reason. Must be without further calculation.

					Examiner's Comments This was a good discriminator as it really tested whether candidates understood how the trapezium rule estimates area. Some believed that it always underestimated or always overestimated.
			Total	5	
2		a	A3*π oe	B1(AO2.2a) [1]	Or 0.125×π oe
		b	$\frac{1}{2}$D2 + D3 + D4 + D5 + $\frac{1}{2}$D6	B1(AO2.2a) [1]	Or equivalent expressed in words.
		c	5.0766 × 0.3927 = 1.9935... 1.99 (units^2) (to 3sf)	M1(AO1.1) A1(AO1.1) [2]	Or 5.0766 × $\frac{\pi}{8}$
			Total	4	
3		i	$T_4 < T_2$ The approximation is an over-estimate, as the trapezia are above the curve therefore the error becomes less when the number of strips increases	B1 B1 [2]	oe (e.g. 4 T_4 is less than T_2) Must see mention of 'over-estimate' and 'above' and 'increasing strips' **Examiner's Comments** The first mark in part (i) was awarded to the vast majority of candidates for correctly stating that T_4 was less than T_2 although some candidates did not make it explicitly clear which value of the two was the least. Candidates found the second mark a lot harder to come by as it was not sufficient to simply state that the approximations given by the trapezium rule were an over-estimate. Candidates needed to make it clear that these approximations were an over-

					estimate because the tops of the trapezia are above the curve which would then, in turn, mean that the error (in using a trapezium rule approximation for the value of the integral) would become less when the number of strips increases.
		ii	0.5×0.5 $\{1.0655\ldots + 2.8963\ldots + 2(1.1695\ldots$ $+ 1.4142\ldots + 1.9282\ldots)\}$ $T_4 = 3.25$ **OR** $0.5587\ldots + 0.6459\ldots + 0.8356\ldots$ $+ 1.2061\ldots$	B1 M1 A1 A1 [4]	For using 0.25 oe The M mark requires the correct {…} bracket structure. It needs the first bracket to contain the first *y* value plus the last *y* value and the second bracket to be multiplied by 2 and to be the summation of the remaining *y* values with no additional values. Allow an error in one value **or** the omission of one value from the second bracket M0 if using *x* values. All values given to at least 3sf or exact The A mark is for the correct {…} bracket with no errors (12.98… or 13.0 implies M1A1) cao (3.25 with no working scores 0/4) – must be given to 2dp only (for reference correct answer is 3.2465079…) SC: bracketing error 0.25 × (1.0655 + 2.8963) + 2(1.1695… + …) scores B1M1A0A0 unless the final answer implies the correct calculation. An answer of 10.014… usually indicates this error Separate trapezia B1 – one area correct (implies 0.25) M1 – three correct (equivalent to one error) A1 – all four correct A1 – cao of 3.25 **Examiner's Comments** Part (ii) was answered extremely well with the vast majority of candidates giving the correct answer of 3.25. When errors occurred it was usually due to an incorrect value for the width of the strips or with the omission of a value. It was very rare for candidates to use the *x* values or to not give the answer to the required 3 significant figures.
			Total	6	

4		a	$$f'(x)=\frac{1}{3}\left(27-8x^3\right)^{-2/3}\times(-24x^2)$$ $$\left[=\frac{-8x^2}{\left(27-8x^3\right)^{2/3}}\right]$$ $f'(1.5)=-\frac{8\times1.5^2}{0}$ and dividing by zero gives the error.	M1 (AO1.1a) A1(AO 1.1) E1 (AO2.4) [3]	Using the chain rule Allow unsimplified Sufficient to say "can't divide by zero" oe **Examiner's Comments** This was well answered as most candidates used the chain rule successfully and realised that substituting $x = 1.5$ gives a zero in the denominator
		b	$$\left(27-8x^3\right)^{1/3}=27^{1/3}\left(1-\frac{8}{27}x^3\right)^{1/3}$$ $$=3\left(1+\left(\frac{1}{3}\right)\left(-\frac{8x^3}{27}\right)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(-\frac{8x^3}{27}\right)^2+\dots\right)$$ $$=3-\frac{8x^3}{27}-\frac{64x^6}{2187}+\dots$$	B1 (AO 3.1a) M1 (AO 1.1a) A1 (AO 1.1b) [1]	Dealing with the 27 correctly Using the Binomial expansion substantially correctly Cao **Examiner's Comments** Many candidates dealt successfully with the 27, but when that was done without clear working shown, could cost 2 marks here. Some candidates simplified their

					coefficients early and incorrectly, so it was not always clear that the binomial expansion had been used, costing the method mark. AfL Make your method clear by writing down the factors of each term before simplifying.
		c	The binomial expansion is valid for $\left\lvert -8\frac{x^3}{27} \right\rvert < 1$ $\lvert x \rvert < 1.5$ and the limits of the integral are completely in this interval.	B1 (AO 2.4) E1 (AO 2.3) [2]	Allow unsimplified but must use correct modulus notation or equivalent Must indicate that the limits of the integral lie in their interval for which the expansion is valid. Examiner's Comments One of the assessment objectives in the specification is to test the ability of a candidate to assess the validity of an argument as in this question. Not many candidates realised that the key to this explanation was to find the range of values for which the binomial expansion is valid. The limits lie well within the valid range so the method is valid.
		d	$\frac{0.25}{2}\left(3+2.6684+2\left(2.9954+2.9625+2.8694\right)\right)$	B1 (AO 1.1a) M1 (AO 1.1b) A1 (AO 1.1b) [3]	$h = 0.25$ used For sum in the bracket – condone one slip. Values from candidates own calculators may differ in the last decimal place.

			$= \frac{0.25}{2} \times 23.3224 = 2.9153$		Allow for 2.92 or better **Examiner's Comments** This was generally done very well.
		e	There is area between the curve and the top of the trapezia, so some area is missing from the estimate.	E1 (AO 2.4) [1]	Allow for any sensible explanation eg the trapezia are under the curve "The curve is concave downwards" on its own is not quite enough **Examiner's Comments** Most candidates were able to explain this clearly. Some had learned that a curve being concave downwards would give an underestimate but gave no indication as to why that would be, so lost the mark. AfL Clear annotated sketches can support a mathematical explanation better than an extended written response.
			Total	12	
5		a	$1 - 0.5^2$	B1 (AO 2.2a) [1]	oe
		b	$\frac{0.5}{2}\left\{\left(2.718282+0.049787\right)+2\left(2.117+1+0.286505\right)\right\}$	M1 (AO 1.1a) A1 (AO 1.1)	

			= 2.39 (correct to 3 significant figures)	[2]	
		c	$\frac{dy}{dx} = -2xe^{1-x^2}$ $\frac{dy}{dx} = 0$ only when $x = 0$, giving $y = e^{1-0} = e$	M1 (AO 1.1a) E1 (AO 2.1) [2]	**AG** Convincing completion
			Total	5	
6		a	$h = 0.5 \Rightarrow$ Integral $\approx \frac{1}{2} \times 0.5 \times 16.2075$ = 4.051 875	M1 (AO 1.1a) A1 (AO 1.1b) [2]	Substitution of h and the total from spreadsheet and using it in the trapezium rule formula awrt 4.05 Allow recalculation of the spreadsheet total from scratch
		b	The estimate is an overestimate; as the curve is concave upwards the tops of the trapezia are above the curve and so the trapezia include extra area	E1 (AO 2.2a) [1]	Overestimate stated with clear explanation (must include reference to trapezia being above the curve, or a suitable diagram showing this) Do not allow for argument based on a value for the integral found by calculator
			Total	3	