Questions

Q1.

$$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4]

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

(Total for question = 6 marks)

Q2.

The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

(3)

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(2)

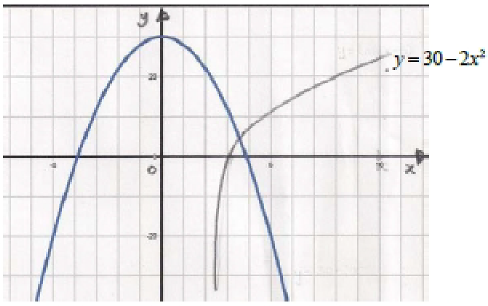
(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

(Total for question = 6 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$f(3.5) = -4.8,\ f(4) = (+)3.1$	M1	1.1b
	Change of sign and function continuous in interval $[3.5, 4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	Attempts $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow \quad x_1 = 4 - \frac{3.099}{16.67}$	M1	1.1b
	$x_1 = 3.81$	A1	1.1b
	$y = \ln(2x - 5)$	(2)	
(c)	 Attempts to sketch both $y = \ln(2x - 5)$ and $y = 30 - 2x^2$	M1	3.1a
	States that $y = \ln(2x - 5)$ meets $y = 30 - 2x^2$ in just one place, therefore $y = \ln(2x - 5) = 30 - 2x$ has just one root $\Rightarrow f(x) = 0$ has just one root	A1	2.4
		(2)	
			(6 marks)

Notes:

(a)

M1: Attempts f(x) at both $x = 3.5$ and $x = 4$ with at least one correct to 1 significant figure

A1*: $f(3.5)$ and f(4) correct to 1 sig figure (rounded or truncated) with a correct reason and conclusion. A reason could be change of sign, or $f(3.5) \times f(4) < 0$ or similar with f(x) being continuous in this interval. A conclusion could be 'Hence root' or 'Therefore root in interval'

(b)

M1: Attempts $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ evidenced by $x_1 = 4 - \frac{3.099}{16.67}$

A1: Correct answer only $x_1 = 3.81$

(c)

M1: For a valid attempt at showing that there is only one root. This can be achieved by

- Sketching graphs of $y = \ln(2x - 5)$ and $y = 30 - 2x^2$ on the same axes
- Showing that $f(x) = \ln(2x - 5) + 2x^2 - 30$ has no turning points
- Sketching a graph of $f(x) = \ln(2x - 5) + 2x^2 - 30$

A1: Scored for correct conclusion

Q2.

Question	Scheme	Marks	AOs
	The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root		
(a)	$\{f(x) = 2x^3 + x^2 - 1 \Rightarrow\}\ f'(x) = 6x^2 + 2x$	B1	1.1b
	$\left\{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} \Rightarrow\right\}\ \{x_{n+1}\} = x_n - \dfrac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$	M1	1.1b
	$= \dfrac{x_n(6x_n^2 + 2x_n) - (2x_n^3 + x_n^2 - 1)}{6x_n^2 + 2x_n} \Rightarrow x_{n+1} = \dfrac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$ *	A1*	2.1
		(3)	
(b)	$\{x_1 = 1 \Rightarrow\}\ x_2 = \dfrac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or $x_2 = 1 - \dfrac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}$	M1	1.1b
	$\Rightarrow x_2 = \dfrac{3}{4},\ x_3 = \dfrac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson **method** cannot be used with $x_1 = 0$ which refer or ***allude*** to either the stationary point or the tangent. E.g. • There is a stationary point at $x = 0$ • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the x-axis • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal	B1	2.3
		(1)	
			(6 marks)

Notes for Question	
(a)	
B1:	States that $f'(x)=6x^2+2x$ or states that $f'(x_n)=6x_n^{\ 2}+2x_n$ (Condone $\frac{dy}{dx}=6x^2+2x$)
M1:	Substitutes $f(x_n)=2x_n^{\ 3}+x_n^{\ 2}-1$ and their $f'(x_n)$ into $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$
A1*:	A correct intermediate step of making a common denominator which leads to the given answer
Note:	Allow B1 if $f'(x)=6x^2+2x$ is applied as $f'(x_n)$ (or $f'(x)$) in the NR formula $\{x_{n+1}\}=x_n-\frac{f(x_n)}{f'(x_n)}$
Note:	Allow M1A1 for • $x_{n+1}=x-\frac{2x^3+x^2-1}{6x^2+2x}=\frac{x(6x^2+2x)-(2x^3+x^2-1)}{6x^2+2x} \Rightarrow x_{n+1}=\frac{4x_n^{\ 3}+x_n^{\ 2}+1}{6x_n^{\ 2}+2x_n}$
Note	Condone $x=x-\frac{2x^3+x^2-1}{"6x^2+2x"}$ for M1
Note	Condone $x_n-\frac{2x_n^{\ 3}+x_n^{\ 2}-1}{"6x_n^{\ 2}+2x_n"}$ or $x-\frac{2x^3+x^2-1}{"6x^2+2x"}$ (i.e. no $x_{n+1}=...$) for M1
Note:	Give M0 for $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$ followed by $x_{n+1}=2x_n^{\ 3}+x_n^{\ 2}-1-\frac{2x_n^{\ 3}+x_n^{\ 2}-1}{6x_n^{\ 2}+2x_n}$
Note:	Correct notation, i.e. x_{n+1} and x_n must be seen in their final answer for A1*
(b)	
M1:	An attempt to use the given or their formula once. Can be implied by $\frac{4(1)^3+(1)^2+1}{6(1)^2+2(1)}$ or 0.75 o.e.
Note:	Allow one slip in substituting $x_1=1$
A1:	$x_2=\frac{3}{4}$ and $x_3=\frac{2}{3}$
Note:	Condone $x_2=\frac{3}{4}$ and $x_3=$ awrt 0.667 for A1
Note:	Condone $\frac{3}{4}, \frac{2}{3}$ listed in a correct order ignoring subscripts
(c)	
B1:	See scheme
Note:	Give B0 for the following isolated reasons: e.g. • You cannot divide by 0 • The fraction (or the NR formula) is undefined at $x=0$ • At $x=0$, $f'(x_1)=0$ • x_1 cannot be 0 • $6x^2+2x$ cannot be 0 • the denominator is 0 which cannot happen • if $x_1=0$, $6x^2+2x=0$