

Numerical Methods Questions

6 [Figure 1, printed on the insert, is provided for use in this question.]

The curve $y = x^3 + 4x - 3$ intersects the x -axis at the point A where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.0. (2 marks)

(b) Show that the equation $x^3 + 4x - 3 = 0$ can be rearranged into the form $x = \frac{3 - x^3}{4}$. (1 mark)

(c) (i) Use the iteration $x_{n+1} = \frac{3 - x_n^3}{4}$ with $x_1 = 0.5$ to find x_3 , giving your answer to two decimal places. (3 marks)

(ii) The sketch on **Figure 1** shows parts of the graphs of $y = \frac{3 - x^3}{4}$ and $y = x$, and the position of x_1 .

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x -axis. (3 marks)

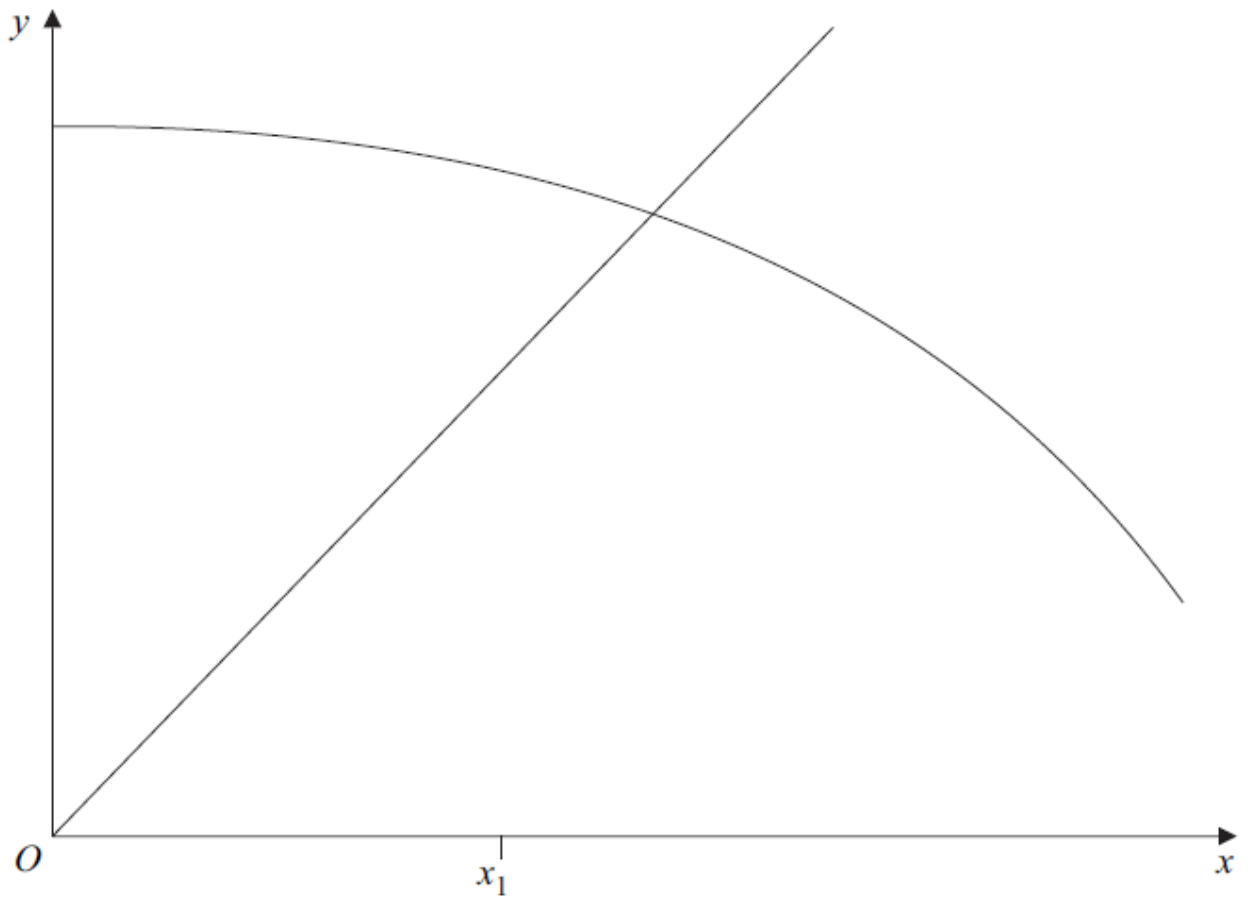
1 The curve $y = x^3 - x - 7$ intersects the x -axis at the point where $x = \alpha$.

(a) Show that α lies between 2.0 and 2.1. (2 marks)

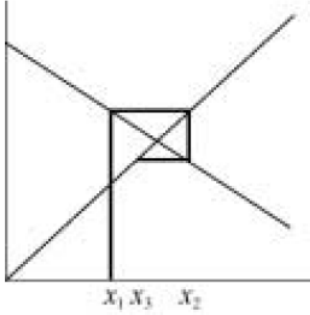
(b) Show that the equation $x^3 - x - 7 = 0$ can be rearranged in the form $x = \sqrt[3]{x + 7}$. (1 mark)

(c) Use the iteration $x_{n+1} = \sqrt[3]{x_n + 7}$ with $x_1 = 2$ to find the values of x_2 , x_3 and x_4 , giving your answers to three significant figures. (3 marks)

Figure 1 (for Question 6)



Numerical Methods Answers

6(a)	$f(0.5) = -0.875$ $f(1) = 2$ Change of sign \therefore root	M1		
		A1	2	
(b)	$x^3 + 4x - 3 = 0$ $4x = 3 - x^3$ $x = \frac{3 - x^3}{4}$	B1	1	AG
(c)(i)	$x_1 = 0.5$ $x_2 = 0.71875$ 0.72 AWRT $x_3 = 0.66$	M1 A1 A1		3
(ii)		M1 A1 A1		3 For cobweb, x_1 to curve For x_2 All correct
Total			9	
1(a)	$f(2) = -1$ $f(2.1) = +0.161$ } change of sign $\therefore 2 < \alpha < 2.1$	M1 A1		2 both attempted
(b)	$x^3 - x - 7 = 0$ $x^3 = x + 7$ $x = \sqrt[3]{x+7}$	B1	1	AG
(c)	$x_1 = 2$ $x_2 = 2.0801\dots$ $x_3 = 2.0862\dots$ $x_4 = 2.09$	M1 A1 A1 A1		3 AWRT 2.08 AWRT 2.09
Total			6	