INTEGRATION

1 **a** Express $\dfrac{1}{x^2 - 3x + 2}$ in partial fractions. **(3)**

b Show that

$$\int_3^4 \frac{1}{x^2 - 3x + 2} \, dx = \ln \frac{a}{b},$$

where a and b are integers to be found. **(5)**

2 Evaluate

$$\int_0^{\frac{\pi}{6}} \cos x \cos 3x \, dx.$$ **(6)**

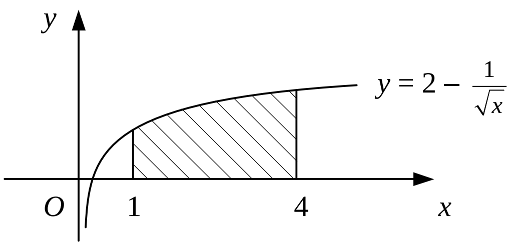
3 **a** Find the quotient and remainder obtained in dividing $(x^2 + x - 1)$ by $(x - 1)$. **(3)**

b Hence, show that

$$\int \frac{x^2 + x - 1}{x - 1} \, dx = \tfrac{1}{2}x^2 + 2x + \ln|x - 1| + c,$$

where c is an arbitrary constant. **(2)**

4



The diagram shows the curve with equation $y = 2 - \dfrac{1}{\sqrt{x}}$.

The shaded region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$ is rotated through 360° about the x-axis to form the solid S.

a Show that the volume of S is $2\pi(2 + \ln 2)$. **(6)**

S is used to model the shape of a container with 1 unit on each axis representing 10 cm.

b Find the volume of the container correct to 3 significant figures. **(2)**

5 **a** Use integration by parts to find $\int x \ln x \, dx$. **(4)**

b Given that $y = 4$ when $x = 2$, solve the differential equation

$$\frac{dy}{dx} = xy \ln x, \quad x > 0, \quad y > 0,$$

and hence, find the exact value of y when $x = 1$. **(5)**

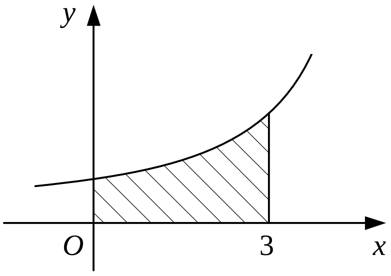
6 **a** Evaluate $\int_0^{\frac{\pi}{3}} \sin x \sec^2 x \, dx$. **(4)**

b Using the substitution $u = \cos\theta$, or otherwise, show that

$$\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\cos^4\theta} \, d\theta = a + b\sqrt{2},$$

where a and b are rational. **(6)**

7



The diagram shows part of the curve with parametric equations

$$x = 2t + 1, \quad y = \frac{1}{2-t}, \quad t \neq 2.$$

The shaded region is bounded by the curve, the coordinate axes and the line $x = 3$.

a Find the value of the parameter t at the points where $x = 0$ and where $x = 3$. **(2)**

b Show that the area of the shaded region is $2 \ln \frac{5}{2}$. **(5)**

c Find the exact volume of the solid formed when the shaded region is rotated completely about the x-axis. **(5)**

8 **a** Using integration by parts, find

$$\int 6x \cos 3x \; dx.$$ **(5)**

b Use the substitution $x = 2 \sin u$ to show that

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} \; dx = \frac{\pi}{3}.$$ **(5)**

9 In an experiment to investigate the formation of ice on a body of water, a thin circular disc of ice is placed on the surface of a tank of water and the surrounding air temperature is kept constant at −5°C.

In a model of the situation, it is assumed that the disc of ice remains circular and that its area, A cm^2 after t minutes, increases at a rate proportional to its perimeter.

a Show that

$$\frac{dA}{dt} = k\sqrt{A},$$

where k is a positive constant. **(3)**

b Show that the general solution of this differential equation is

$$A = (pt + q)^2,$$

where p and q are constants. **(4)**

Given that when $t = 0$, $A = 25$ and that when $t = 20$, $A = 40$,

c find how long it takes for the area to increase to 50 cm^2. **(5)**

10

$$f(x) \equiv \frac{5x+1}{(1-x)(1+2x)}.$$

a Express f(x) in partial fractions. **(3)**

b Find $\int_0^{\frac{1}{2}} f(x) \; dx$, giving your answer in the form $k \ln 2$. **(4)**

c Find the series expansion of f(x) in ascending powers of x up to and including the term in x^3, for $|x| < \frac{1}{2}$. **(6)**