# INTEGRATION

**1** Show that

$$\int_2^7 \frac{8}{4x-3}\,\mathrm{d}x = \ln 25. \qquad \textbf{(4)}$$

**2** Given that $y = \frac{\pi}{4}$ when $x = 1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x \sec y \operatorname{cosec}^3 y. \qquad \textbf{(7)}$$

**3** **a** Use the trapezium rule with three intervals of equal width to find an approximate value for the integral

$$\int_0^{1.5} \mathrm{e}^{x^2-1}\,\mathrm{d}x. \qquad \textbf{(4)}$$

**b** Use the trapezium rule with six intervals of equal width to find an improved approximation for the above integral. **(2)**

**4**
$$\mathrm{f}(x) \equiv \frac{3(2-x)}{(1-2x)^2(1+x)}.$$

**a** Express f($x$) in partial fractions. **(4)**

**b** Show that

$$\int_1^2 \mathrm{f}(x)\,\mathrm{d}x = 1 - \ln 2. \qquad \textbf{(6)}$$

**5** The rate of growth in the number of yeast cells, $N$, present in a culture after $t$ hours is proportional to $N$.

**a** By forming and solving a differential equation, show that

$$N = A\mathrm{e}^{kt},$$

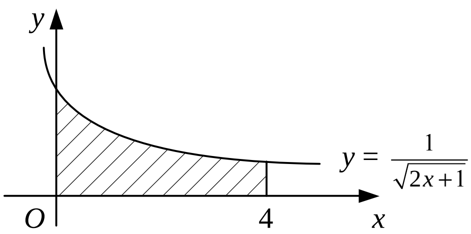
where $A$ and $k$ are positive constants. **(4)**

Initially there are 200 yeast cells in the culture and after 2 hours there are 3000 yeast cells in the culture. Find, to the nearest minute, after how long

**b** there are 10 000 yeast cells in the culture, **(5)**

**c** the number of yeast cells is increasing at the rate of 5 per second. **(4)**

**6**




The diagram shows part of the curve with equation $y = \frac{1}{\sqrt{2x+1}}$.

The shaded region is bounded by the curve, the coordinate axes and the line $x = 4$.

**a** Find the area of the shaded region. **(4)**

The shaded region is rotated through four right angles about the $x$-axis.

**b** Find the volume of the solid formed, giving your answer in the form $\pi \ln k$. **(5)**

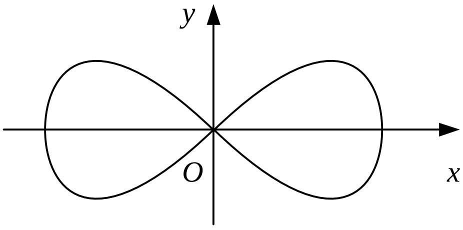
**7** Using the substitution $u^2 = x + 3$, show that

$$\int_0^1 x\sqrt{x+3}\ \mathrm{d}x = k(3\sqrt{3} - 4),$$

where $k$ is a rational number to be found. **(7)**

**8** **a** Use the identities for $\sin(A + B)$ and $\sin(A - B)$ to prove that

$$2\sin A\cos B \equiv \sin(A + B) + \sin(A - B).$$ **(2)**




The diagram shows the curve given by the parametric equations

$$x = 2\sin 2t, \quad y = \sin 4t, \quad 0 \le t < \pi.$$

**b** Show that the total area enclosed by the two loops of the curve is given by

$$\int_0^{\frac{\pi}{4}} 16\sin 4t\cos 2t\ \mathrm{d}t.$$ **(4)**

**c** Evaluate this integral. **(5)**

**9** $$\mathrm{f}(x) \equiv \frac{x^2 - 22}{(x+2)(x-4)}.$$

**a** Find the values of the constants $A$, $B$ and $C$ such that

$$\mathrm{f}(x) \equiv A + \frac{B}{x+2} + \frac{C}{x-4}.$$ **(3)**

The finite region $R$ is bounded by the curve $y = \mathrm{f}(x)$, the coordinate axes and the line $x = 2$.

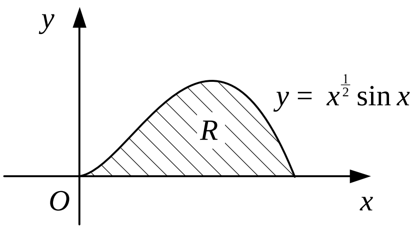
**b** Find the area of $R$, giving your answer in the form $p + \ln q$, where $p$ and $q$ are integers. **(5)**

**10** **a** Find $\int \sin^2 x\ \mathrm{d}x$. **(4)**

**b** Use integration by parts to show that

$$\int x\sin^2 x\ \mathrm{d}x = \tfrac{1}{8}(2x^2 - 2x\sin 2x - \cos 2x) + c,$$

where $c$ is an arbitrary constant. **(4)**




The diagram shows the curve with equation $y = x^{\frac{1}{2}}\sin x$, $0 \le x \le \pi$.

The finite region $R$, bounded by the curve and the $x$-axis, is rotated through $2\pi$ radians about the $x$-axis.

**c** Find the volume of the solid formed, giving your answer in terms of $\pi$. **(3)**