INTEGRATION — *Answers*

1 **a**

x	1	3	5
$x \ln (x+1)$	$\ln 2$	$3 \ln 4$	$5 \ln 6$

$\therefore$ integral $\approx \frac{1}{2} \times 2 \times [\ln 2 + 5 \ln 6 + 2(3 \ln 4)] = 18.0$ (3sf)

b

x	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cot x$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	0

$\therefore$ integral $\approx \frac{1}{2} \times \frac{\pi}{6} \times [\sqrt{3} + 0 + 2(\frac{1}{\sqrt{3}})] = 0.756$ (3sf)

c

x	−2	−1	0	1	2
$e^{\frac{x^2}{10}}$	1.492	1.105	1	1.105	1.492

$\therefore$ integral $\approx \frac{1}{2} \times 1 \times [1.492 + 1.492 + 2(1.105 + 1 + 1.105)] = 4.70$ (3sf)

d

x	0	0.25	0.5	0.75	1
$\arccos (x^2 - 1)$	3.142	2.786	2.419	2.024	1.571

$\therefore$ integral $\approx \frac{1}{2} \times 0.25 \times [3.142 + 1.571 + 2(2.786 + 2.419 + 2.024)] = 2.40$ (3sf)

e

x	0	0.1	0.2	0.3	0.4	0.5
$\sec^2 (2x - 1)$	3.4255	2.0602	1.4680	1.1788	1.0411	1

$\therefore$ integral $\approx \frac{1}{2} \times 0.1 \times [3.4255 + 1 + 2(2.0602 + 1.4680 + 1.1788 + 1.0411)] = 0.796$ (3sf)

f

x	0	1	2	3	4	5	6
$x^3 e^{-x}$	0	0.368	1.083	1.344	1.172	0.842	0.535

$\therefore$ integral $\approx \frac{1}{2} \times 1 \times [0 + 0.535 + 2(0.368 + 1.083 + 1.344 + 1.172 + 0.842)] = 5.08$ (3sf)

2 **a** $2 - \frac{1}{\sin x} = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

b

x	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$
$2 - \operatorname{cosec} x$	0	0.8453	1	0.8453	0

$\therefore$ area $\approx \frac{1}{2} \times \frac{\pi}{6} \times [0 + 0 + 2(0.8453 + 1 + 0.8453)] = 1.41$ (3sf)

3 **a**

x	−1	0	1	2
f(x)	0	0.5236	1.0472	2.0944

$\therefore$ $I \approx \frac{1}{2} \times 1 \times [0 + 2.0944 + 2(0.5236 + 1.0472)] = 2.62$ (3sf)

b

x	−1	−0.5	0	0.5	1	1.5	2
f(x)	0	0.2709	0.5236	0.7763	1.0472	1.3717	2.0944

$\therefore$ $I \approx \frac{1}{2} \times 0.5 \times [0 + 2.0944 + 2(0.2709 + 0.5236 + 0.7763 + 1.0472 + 1.3717)] = 2.52$ (3sf)

4 **a**

x	1	1.5	2	2.5	3	3.5	4	4.5	5
$\ln x$	0	$\ln 1.5$	$\ln 2$	$\ln 2.5$	$\ln 3$	$\ln 3.5$	$\ln 4$	$\ln 4.5$	$\ln 5$

i $\approx \frac{1}{2} \times 2 \times [0 + \ln 5 + 2(\ln 3)] = 3.807$ (3dp)

ii $\approx \frac{1}{2} \times 1 \times [0 + \ln 5 + 2(\ln 2 + \ln 3 + \ln 4)] = 3.983$ (3dp)

iii $\approx \frac{1}{2} \times 0.5 \times [0 + \ln 5 + 2(\ln 1.5 + \ln 2 + \ln 2.5 + \ln 3 + \ln 3.5 + \ln 4 + \ln 4.5)] = 4.031$ (3dp)

b $2 \rightarrow 4$ strips, increase = 0.176

$4 \rightarrow 8$ strips, increase = 0.048

e.g. suggest $8 \rightarrow 16$ strips, increase ≈ 0.013

$16 \rightarrow 32$ strips, increase ≈ 0.004

$32 \rightarrow 64$ strips, increase ≈ 0.001

$\therefore$ area $\approx 4.031 + 0.013 + 0.004 + 0.001 = 4.049$

c $u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$; $\frac{dv}{dx} = 1$, $v = x$

$$\int_1^5 \ln x \, dx = [x \ln x]_1^5 - \int_1^5 \frac{1}{x} \times x \, dx$$

$$= [x \ln x - x]_1^5$$

$$= (5 \ln 5 - 5) - (0 - 1)$$

$$= 5 \ln 5 - 4$$

$$= 4.047 \text{ (3dp)}$$

5 volume $= \pi \int_{-4}^{0} (e^x - x)^2 \, dx$

let $I = \int_{-4}^{0} (e^x - x)^2 \, dx$

x	-4	-3	-2	-1	0
$(e^x - x)^2$	16.147	9.301	4.560	1.871	1

$\therefore$ $I \approx \frac{1}{2} \times 1 \times [16.147 + 1 + 2(9.301 + 4.560 + 1.871)] = 24.306$

$\therefore$ volume $\approx 24.306 \times \pi = 76.4$ (3sf)