> **INTEGRATION** *Answers*

1 **a**

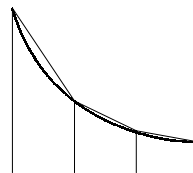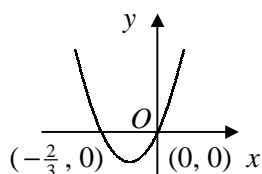| x	1	2	3	4
y	3	$\frac{3}{2}$	1	$\frac{3}{4}$

b $= \frac{1}{2} \times 1 \times [3 + \frac{3}{4} + 2(\frac{3}{2} + 1)] = 4\frac{3}{8}$

c the true area is less
the curve passes below the top of each trapezium as shown:

2 **a**

$(-\frac{2}{3}, 0)$ $(0, 0)$ x

b

x	0	0.5	1	1.5	2
$x(3x + 2)$	0	1.75	5	9.75	16

area $\approx \frac{1}{2} \times 0.5 \times [0 + 16 + 2(1.75 + 5 + 9.75)] = 12.25$

c $= \int_0^2 (3x^2 + 2x) \, dx = [x^3 + x^2]_0^2 = (8 + 4) - 0 = 12$

d % error $= \frac{12.25 - 12}{12} \times 100\% = 2.08\%$ (3sf)

3 **a**

x	4	5	6
$\frac{3}{2x+1}$	$\frac{1}{3}$	$\frac{3}{11}$	$\frac{3}{13}$

∴ area $\approx \frac{1}{2} \times 1 \times [\frac{1}{3} + \frac{3}{13} + 2(\frac{3}{11})] = 0.555$ (3sf)

b

x	0	1	2	3
$\lg(x^2 + 9)$	$\lg 9$	$\lg 10$	$\lg 13$	$\lg 18$

∴ area $\approx \frac{1}{2} \times 1 \times [\lg 9 + \lg 18 + 2(\lg 10 + \lg 13)] = 3.22$ (3sf)

c

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$x^2 \sin x$	0	0.436	2.467	3.926	0

∴ area $\approx \frac{1}{2} \times \frac{\pi}{4} \times [0 + 0 + 2(0.436 + 2.467 + 3.926)] = 5.36$ (3sf)

d

x	−2	−1	0	1	2
$\sqrt[3]{2x+5}$	1	$\sqrt[3]{3}$	$\sqrt[3]{5}$	$\sqrt[3]{7}$	$\sqrt[3]{9}$

∴ area $\approx \frac{1}{2} \times 1 \times [1 + \sqrt[3]{9} + 2(\sqrt[3]{3} + \sqrt[3]{5} + \sqrt[3]{7})] = 6.61$ (3sf)

4 **a**

x	0	1	2	3
3^x	1	3	9	27

∴ area $\approx \frac{1}{2} \times 1 \times [1 + 27 + 2(3 + 9)] = 26$

b

x	2	2.2	2.4
$\sin(\lg x)$	0.2965	0.3358	0.3711

∴ area $\approx \frac{1}{2} \times 0.2 \times [0.2965 + 0.3711 + 2(0.3358)] = 0.134$ (3sf)

c

x	0	0.1	0.2	0.3	0.4	0.5
$\dfrac{x}{x^3+2}$	0	0.04998	0.09960	0.14800	0.19380	0.23529

$\therefore$ area $\approx \frac{1}{2} \times 0.1 \times [0+0.23529+2(0.04998+0.09960+0.14800+0.19380)] = 0.0609$ (3sf)

d

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$\sqrt{\cos\left(\frac{1}{2}x\right)}$	1	0.9828	0.9306	0.8409	0.7071

$\therefore$ area $\approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 0.7071 + 2(0.9828 + 0.9306 + 0.8409)] = 1.89$ (3sf)

5 a

x	3	3.8	4.6	5.4	6.2	7
$2 - 3x^{-\frac{1}{2}}$	0.2679	0.4610	0.6012	0.7090	0.7952	0.8661

area $\approx \frac{1}{2} \times 0.8 \times [0.2679 + 0.8661 + 2(0.4610 + 0.6012 + 0.7090 + 0.7952)] = 2.51$ (3sf)

b $= \displaystyle\int_3^7 (2 - 3x^{-\frac{1}{2}}) \ dx$

$= [2x - 6x^{\frac{1}{2}}]_3^7 = (14 - 6\sqrt{7}) - (6 - 6\sqrt{3}) = 8 + 6(\sqrt{3} - \sqrt{7})$

6 a

x	0	0.2	0.4	0.6	0.8	1
$\sin x^2$	0	0.03999	0.15932	0.35227	0.59720	0.84147

area $\approx \frac{1}{2} \times 0.2 \times [0 + 0.84147 + 2(0.03999 + 0.15932 + 0.35227 + 0.59720)] = 0.3139$

b area of rectangle $= 1 \times 0.8415 = 0.8415$

$\therefore$ area of flower bed $\approx 0.8415 - 0.3139 = 0.53$ m^2

7 a $= 1 + 6(\frac{x}{2}) + \frac{6\times5}{2}(\frac{x}{2})^2 + \frac{6\times5\times4}{3\times2}(\frac{x}{2})^3 + \ldots$

$= 1 + 3x + \frac{15}{4}x^2 + \frac{5}{2}x^3 + \ldots$

b area $\approx \displaystyle\int_0^{0.5} (1 + 3x + \frac{15}{4}x^2 + \frac{5}{2}x^3) \ dx$

$= [x + \frac{3}{2}x^2 + \frac{5}{4}x^3 + \frac{5}{8}x^4]_0^{0.5}$

$= (\frac{1}{2} + \frac{3}{8} + \frac{5}{32} + \frac{5}{128}) - 0 = 1.07$ (3sf)

c

x	0	0.1	0.2	0.3	0.4	0.5
$(1 + \frac{x}{2})^6$	1	1.3401	1.7716	2.3131	2.9860	3.8147

area $\approx \frac{1}{2} \times 0.1 \times [1 + 3.8147 + 2(1.3401 + 1.7716 + 2.3131 + 2.9860)] = 1.08$ (3sf)

8 a $\dfrac{dy}{dx} = 2x - 16x^{-2}$

SP: $2x - 16x^{-2} = 0$

      $x^3 = 8$

      $x = 2$

when $x = 2$, $y = 4 + 8 = 12$ $\therefore$ SP (2, 12)

b

x	2	2.5	3	3.5	4
$x^2 + \dfrac{16}{x}$	12	12.65	14.333	16.821	20

area $\approx \frac{1}{2} \times 0.5 \times [12 + 20 + 2(12.65 + 14.333 + 16.821)] = 29.9$ (3sf)

c over-estimate