INTEGRATION	*Answers*

1   **a** $x = 0 \Rightarrow t = 2$

$x = 2 \Rightarrow t = 3$

b area $= \int_0^2 y \; dx$

$x = 2t - 4 \; \therefore \; \dfrac{dx}{dt} = 2$

$\therefore$ area $= \int_2^3 \dfrac{1}{t} \times 2 \; dt$

$= \int_2^3 \dfrac{2}{t} \; dt$

c $= [2 \ln |t| \,]_2^3$

$= 2 \ln 3 - 2 \ln 2$

$= 2 \ln \tfrac{3}{2}$

d $t = \dfrac{x+4}{2}$

$\therefore \; y = \dfrac{2}{x+4}$

$\therefore$ area $= \int_0^2 \dfrac{2}{x+4} \; dx$

$= [2 \ln |x+4| \,]_0^2$

$= 2 \ln 6 - 2 \ln 4$

$= 2 \ln \tfrac{3}{2}$

2   **a** $x = 0 \Rightarrow \cos \theta = 0 \Rightarrow \theta = \tfrac{\pi}{2}, \tfrac{3\pi}{2}$

for $y > 0$, $\theta = \tfrac{\pi}{2}$ at A

$y = 0 \Rightarrow \sin \theta = 0 \Rightarrow \theta = 0, \pi$

for $x > 0$, $\theta = 0$ at B

b $x = 4 \cos \theta \; \therefore \; \dfrac{dx}{d\theta} = -4 \sin \theta$

$\therefore$ area $= \int_{\frac{\pi}{2}}^0 2 \sin \theta \times -4 \sin \theta \; d\theta$

$= \int_0^{\frac{\pi}{2}} 8 \sin^2 \theta \; d\theta$

c shaded area $= \int_0^{\frac{\pi}{2}} (4 - 4 \cos 2\theta) \; d\theta$

$= [4\theta - 2 \sin 2\theta \,]_0^{\frac{\pi}{2}}$

$= (2\pi - 0) - (0 - 0)$

$= 2\pi$

area of ellipse $= 4 \times 2\pi$

$= 8\pi$

3   **a** $y = 0 \Rightarrow \sin 2t = 0 \Rightarrow t = 0, \tfrac{\pi}{2}$

$x = 2 \sin t \; \therefore \; \dfrac{dx}{dt} = 2 \cos t$

area above x-axis

$= \int_0^{\frac{\pi}{2}} 5 \sin 2t \times 2 \cos t \; dt$

$= \int_0^{\frac{\pi}{2}} 10 \sin 2t \cos t \; dt$

area enclosed by curve

$= 2 \int_0^{\frac{\pi}{2}} 10 \sin 2t \cos t \; dt$

$= \int_0^{\frac{\pi}{2}} 20 \sin 2t \cos t \; dt$

b $= 40 \int_0^{\frac{\pi}{2}} \sin t \cos^2 t \; dt$

$= -40 \int_0^{\frac{\pi}{2}} (-\sin t) \cos^2 t \; dt$

$= -40 [\tfrac{1}{3} \cos^3 t \,]_0^{\frac{\pi}{2}}$

$= -\tfrac{40}{3} (0 - 1)$

$= 13\tfrac{1}{3}$

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