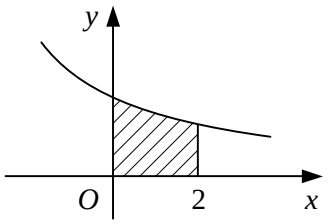
INTEGRATION

1



The diagram shows part of the curve with parametric equations

$$x = 2t - 4, \quad y = \frac{1}{t}.$$

The shaded region is bounded by the curve, the coordinate axes and the line $x = 2$.

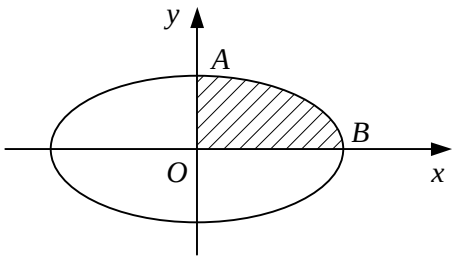
a Find the value of the parameter t when $x = 0$ and when $x = 2$.

b Show that the area of the shaded region is given by $\int_2^3 \frac{2}{t}\ \mathrm{d}t$.

c Hence, find the area of the shaded region.

d Verify your answer to part **c** by first finding a cartesian equation for the curve.

2



The diagram shows the ellipse with parametric equations

$$x = 4\cos\theta, \quad y = 2\sin\theta, \quad 0 \leq \theta < 2\pi,$$

which meets the positive coordinate axes at the points A and B.

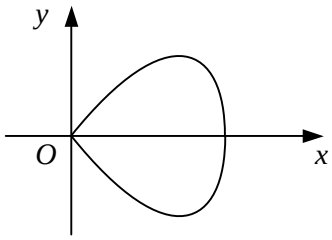
a Find the value of the parameter θ at the points A and B.

b Show that the area of the shaded region bounded by the curve and the positive coordinate axes is given by

$$\int_0^{\frac{\pi}{2}} 8\sin^2\theta\ \mathrm{d}\theta.$$

c Hence, show that the area of the region enclosed by the ellipse is 8π.

3



The diagram shows the curve with parametric equations

$$x = 2\sin t, \quad y = 5\sin 2t, \quad 0 \leq t < \pi.$$

a Show that the area of the region enclosed by the curve is given by $\int_0^{\frac{\pi}{2}} 20\sin 2t\cos t\ \mathrm{d}t$.

b Evaluate this integral.