

INTEGRATION

- 1 a** Express $\frac{3x+5}{(x+1)(x+3)}$ in partial fractions.
- b** Hence, find $\int \frac{3x+5}{(x+1)(x+3)} dx$.
- 2** Show that $\int \frac{3}{(t-2)(t+1)} dt = \ln \left| \frac{t-2}{t+1} \right| + c$.
- 3** Integrate with respect to x
- a** $\frac{6x-11}{(2x+1)(x-3)}$ **b** $\frac{14-x}{x^2+2x-8}$ **c** $\frac{6}{(2+x)(1-x)}$ **d** $\frac{x+1}{5x^2-14x+8}$
- 4 a** Find the values of the constants A , B and C such that
- $$\frac{x^2-6}{(x+4)(x-1)} \equiv A + \frac{B}{x+4} + \frac{C}{x-1}.$$
- b** Hence, find $\int \frac{x^2-6}{(x+4)(x-1)} dx$.
- 5 a** Express $\frac{x^2-x-4}{(x+2)(x+1)^2}$ in partial fractions.
- b** Hence, find $\int \frac{x^2-x-4}{(x+2)(x+1)^2} dx$.
- 6** Integrate with respect to x
- a** $\frac{3x^2-5}{x^2-1}$ **b** $\frac{x(4x+13)}{(2+x)^2(3-x)}$ **c** $\frac{x^2-x+1}{x^2-3x-10}$ **d** $\frac{2-6x+5x^2}{x^2(1-2x)}$
- 7** Show that $\int_3^4 \frac{3x-5}{(x-1)(x-2)} dx = 2 \ln 3 - \ln 2$.
- 8** Find the exact value of
- a** $\int_1^3 \frac{x+3}{x(x+1)} dx$ **b** $\int_0^2 \frac{3x-2}{x^2+x-12} dx$ **c** $\int_1^2 \frac{9}{2x^2-7x-4} dx$
- d** $\int_0^2 \frac{2x^2-7x+7}{x^2-2x-3} dx$ **e** $\int_0^1 \frac{5x+7}{(x+1)^2(x+3)} dx$ **f** $\int_{-1}^1 \frac{2+x}{8-2x-x^2} dx$
- 9 a** Express $\frac{1}{x^2-a^2}$, where a is a positive constant, in partial fractions.
- b** Hence, show that $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$.
- c** Find $\int \frac{1}{a^2-x^2} dx$.
- 10** Evaluate
- a** $\int_{-1}^1 \frac{1}{x^2-9} dx$ **b** $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{4}{1-x^2} dx$ **c** $\int_0^1 \frac{3}{2x^2-8} dx$