

INTEGRATION

- 1 Using integration by parts, show that

$$\int x \cos x \, dx = x \sin x + \cos x + c.$$

- 2 Use integration by parts to find

a $\int x e^x \, dx$

b $\int 4x \sin x \, dx$

c $\int x \cos 2x \, dx$

d $\int x\sqrt{x+1} \, dx$

e $\int \frac{x}{e^{3x}} \, dx$

f $\int x \sec^2 x \, dx$

- 3 Using

i integration by parts,

ii the substitution $u = 2x + 1$,

find $\int x(2x + 1)^3 \, dx$, and show that your answers are equivalent.

- 4 Show that

$$\int_0^2 x e^{-x} \, dx = 1 - 3e^{-2}.$$

- 5 Evaluate

a $\int_0^{\frac{\pi}{6}} x \cos x \, dx$

b $\int_0^1 x e^{2x} \, dx$

c $\int_0^{\frac{\pi}{4}} x \sin 3x \, dx$

- 6 Using integration by parts twice in each case, show that

a $\int x^2 e^x \, dx = e^x(x^2 - 2x + 2) + c,$

b $\int e^x \sin x \, dx = \frac{1}{2} e^x(\sin x - \cos x) + c.$

- 7 Find

a $\int x^2 \sin x \, dx$

b $\int x^2 e^{3x} \, dx$

c $\int e^{-x} \cos 2x \, dx$

- 8 **a** Write down the derivative of $\ln x$ with respect to x .

b Use integration by parts to find

$$\int \ln x \, dx.$$

- 9 Find

a $\int \ln 2x \, dx$

b $\int 3x \ln x \, dx$

c $\int (\ln x)^2 \, dx$

- 10 Evaluate

a $\int_{-1}^0 (x+2)e^x \, dx$

b $\int_1^2 x^2 \ln x \, dx$

c $\int_{\frac{1}{3}}^1 2x e^{3x-1} \, dx$

d $\int_0^3 \ln(2x+3) \, dx$

e $\int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$

f $\int_0^{\frac{\pi}{4}} e^{3x} \sin 2x \, dx$