INTEGRATION

1 Showing your working in full, use the given substitution to find

a $\int 2x(x^2 - 1)^3 \, dx$ $\quad u = x^2 + 1$

b $\int \sin^4 x \cos x \, dx$ $\quad u = \sin x$

c $\int 3x^2(2 + x^3)^2 \, dx$ $\quad u = 2 + x^3$

d $\int 2x e^{x^2} \, dx$ $\quad u = x^2$

e $\int \frac{x}{(x^2+3)^4} \, dx$ $\quad u = x^2 + 3$

f $\int \sin 2x \cos^3 2x \, dx$ $\quad u = \cos 2x$

g $\int \frac{3x}{x^2 - 2} \, dx$ $\quad u = x^2 - 2$

h $\int x\sqrt{1 - x^2} \, dx$ $\quad u = 1 - x^2$

i $\int \sec^3 x \tan x \, dx$ $\quad u = \sec x$

j $\int (x + 1)(x^2 + 2x)^3 \, dx$ $\quad u = x^2 + 2x$

2 **a** Given that $u = x^2 + 3$, find the value of u when

i $x = 0$
ii $x = 1$

b Using the substitution $u = x^2 + 3$, show that

$$\int_0^1 2x(x^2 + 3)^2 \, dx = \int_3^4 u^2 \, du.$$

c Hence, show that

$$\int_0^1 2x(x^2 + 3)^2 \, dx = 12\tfrac{1}{3}.$$

3 Using the given substitution, evaluate

a $\int_1^2 x(x^2 - 3)^3 \, dx$ $\quad u = x^2 - 3$

b $\int_0^{\frac{\pi}{6}} \sin^3 x \cos x \, dx$ $\quad u = \sin x$

c $\int_0^3 \frac{4x}{x^2 + 1} \, dx$ $\quad u = x^2 + 1$

d $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x \sec^2 x \, dx$ $\quad u = \tan x$

e $\int_2^3 \frac{x}{\sqrt{x^2 - 3}} \, dx$ $\quad u = x^2 - 3$

f $\int_{-2}^{-1} x^2(x^3 + 2)^2 \, dx$ $\quad u = x^3 + 2$

g $\int_0^1 e^{2x}(1 + e^{2x})^3 \, dx$ $\quad u = 1 + e^{2x}$

h $\int_3^5 (x - 2)(x^2 - 4x)^2 \, dx$ $\quad u = x^2 - 4x$

4 **a** Using the substitution $u = 4 - x^2$, show that

$$\int_0^2 x(4 - x^2)^3 \, dx = \int_0^4 \tfrac{1}{2}u^3 \, du.$$

b Hence, evaluate

$$\int_0^2 x(4 - x^2)^3 \, dx.$$

5 Using the given substitution, evaluate

a $\int_0^1 x e^{2 - x^2} \, dx$ $\quad u = 2 - x^2$

b $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos x} \, dx$ $\quad u = 1 + \cos x$

6 **a** By writing $\cot x$ as $\frac{\cos x}{\sin x}$, use the substitution $u = \sin x$ to show that

$$\int \cot x \, dx = \ln|\sin x| + c.$$

b Show that

$$\int \tan x \, dx = \ln|\sec x| + c.$$

c Evaluate

$$\int_0^{\frac{\pi}{6}} \tan 2x \, dx.$$

7 By recognising a function and its derivative, or by using a suitable substitution, integrate with respect to x

a $3x^2(x^3 - 2)^3$ **b** $e^{\sin x} \cos x$ **c** $\frac{x}{x^2 + 1}$

d $(2x + 3)(x^2 + 3x)^2$ **e** $x\sqrt{x^2 + 4}$ **f** $\cot^3 x \operatorname{cosec}^2 x$

g $\frac{e^x}{1 + e^x}$ **h** $\frac{\cos 2x}{3 + \sin 2x}$ **i** $\frac{x^3}{(x^4 - 2)^2}$

j $\frac{(\ln x)^3}{x}$ **k** $x^{\frac{1}{2}}(1 + x^{\frac{3}{2}})^2$ **l** $\frac{x}{\sqrt{5 - x^2}}$

8 Evaluate

a $\int_0^{\frac{\pi}{2}} \sin x\,(1 + \cos x)^2 \, dx$ **b** $\int_{-1}^{0} \frac{e^{2x}}{2 - e^{2x}} \, dx$

c $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot x \operatorname{cosec}^4 x \, dx$ **d** $\int_2^4 \frac{x + 1}{x^2 + 2x + 8} \, dx$

9 Using the substitution $u = x + 1$, show that

$$\int x(x + 1)^3 \, dx = \tfrac{1}{20}(4x - 1)(x + 1)^4 + c.$$

10 Using the given substitution, find

a $\int x(2x - 1)^4 \, dx$ $u = 2x - 1$ **b** $\int x\sqrt{1 - x} \, dx$ $u^2 = 1 - x$

c $\int \frac{1}{(1 - x^2)^{\frac{3}{2}}} \, dx$ $x = \sin u$ **d** $\int \frac{1}{\sqrt{x} - 1} \, dx$ $x = u^2$

e $\int (x + 1)(2x + 3)^3 \, dx$ $u = 2x + 3$ **f** $\int \frac{x^2}{\sqrt{x - 2}} \, dx$ $u^2 = x - 2$

11 Using the given substitution, evaluate

a $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1 - x^2}} \, dx$ $x = \sin u$ **b** $\int_0^2 x(2 - x)^3 \, dx$ $u = 2 - x$

c $\int_0^1 \sqrt{4 - x^2} \, dx$ $x = 2\sin u$ **d** $\int_0^3 \frac{x^2}{x^2 + 9} \, dx$ $x = 3\tan u$