

# INTEGRATION

1 Integrate with respect to  $x$

- a**  $2 \cos x$                       **b**  $\sin 4x$                       **c**  $\cos \frac{1}{2}x$                       **d**  $\sin(x + \frac{\pi}{4})$   
**e**  $\cos(2x - 1)$                       **f**  $3 \sin(\frac{\pi}{3} - x)$                       **g**  $\sec x \tan x$                       **h**  $\operatorname{cosec}^2 x$   
**i**  $5 \sec^2 2x$                       **j**  $\operatorname{cosec} \frac{1}{4}x \cot \frac{1}{4}x$                       **k**  $\frac{4}{\sin^2 x}$                       **l**  $\frac{1}{\cos^2(4x+1)}$

2 Evaluate

- a**  $\int_0^{\frac{\pi}{2}} \cos x \, dx$                       **b**  $\int_0^{\frac{\pi}{6}} \sin 2x \, dx$                       **c**  $\int_0^{\frac{\pi}{2}} 2 \sec \frac{1}{2}x \tan \frac{1}{2}x \, dx$   
**d**  $\int_0^{\frac{\pi}{3}} \cos(2x - \frac{\pi}{3}) \, dx$                       **e**  $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2 3x \, dx$                       **f**  $\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \operatorname{cosec} x \cot x \, dx$

3 **a** Express  $\tan^2 \theta$  in terms of  $\sec \theta$ .

**b** Show that  $\int \tan^2 x \, dx = \tan x - x + c$ .

4 **a** Use the identity for  $\cos(A + B)$  to express  $\cos^2 A$  in terms of  $\cos 2A$ .

**b** Find  $\int \cos^2 x \, dx$ .

5 Find

- a**  $\int \sin^2 x \, dx$                       **b**  $\int \cot^2 2x \, dx$                       **c**  $\int \sin x \cos x \, dx$   
**d**  $\int \frac{\sin x}{\cos^2 x} \, dx$                       **e**  $\int 4 \cos^2 3x \, dx$                       **f**  $\int (1 + \sin x)^2 \, dx$   
**g**  $\int (\sec x - \tan x)^2 \, dx$                       **h**  $\int \operatorname{cosec} 2x \cot x \, dx$                       **i**  $\int \cos^4 x \, dx$

6 Evaluate

- a**  $\int_0^{\frac{\pi}{2}} 2 \cos^2 x \, dx$                       **b**  $\int_0^{\frac{\pi}{4}} \cos 2x \sin 2x \, dx$                       **c**  $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \tan^2 \frac{1}{2}x \, dx$   
**d**  $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\cos 2x}{\sin^2 2x} \, dx$                       **e**  $\int_0^{\frac{\pi}{4}} (1 - 2 \sin x)^2 \, dx$                       **f**  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x \operatorname{cosec}^2 x \, dx$

7 **a** Use the identities for  $\sin(A + B)$  and  $\sin(A - B)$  to show that

$$\sin A \cos B \equiv \frac{1}{2} [\sin(A + B) + \sin(A - B)].$$

**b** Find  $\int \sin 3x \cos x \, dx$ .

8 Integrate with respect to  $x$

- a**  $2 \sin 5x \sin x$                       **b**  $\cos 2x \cos x$                       **c**  $4 \sin x \cos 4x$                       **d**  $\cos(x + \frac{\pi}{6}) \sin x$