

INTEGRATION

Answers

- 1** **a** $e^x + c$ **b** $4e^x + c$ **c** $\ln|x| + c$ **d** $6\ln|x| + c$
- 2** **a** $= 2t + 3e^t + c$ **b** $= \frac{1}{2}t^2 + \ln|t| + c$ **c** $= \frac{1}{3}t^3 - e^t + c$ **d** $= 9t - 2\ln|t| + c$
- e** $= \int \left(\frac{7}{t} + t^{\frac{1}{2}}\right) dt$ **f** $= \frac{1}{4}e^t - \ln|t| + c$ **g** $= \int \left(\frac{1}{3t} + t^{-2}\right) dt$ **h** $= \frac{2}{5}\ln|t| - \frac{3}{7}e^t + c$
- $= 7\ln|t| + \frac{2}{3}t^{\frac{3}{2}} + c$ $= \frac{1}{3}\ln|t| - t^{-1} + c$
- 3** **a** $= 5x - 3\ln|x| + c$ **b** $= \ln|u| - u^{-1} + c$ **c** $= \int \left(\frac{2}{5}e^t + \frac{1}{5}\right) dt$
- $= \frac{2}{5}e^t + \frac{1}{5}t + c$
- d** $= \int \left(3 + \frac{1}{y}\right) dy$ **e** $= \int \left(\frac{3}{4}e^t + 3t^{\frac{1}{2}}\right) dt$ **f** $= \int (x^2 - 2 + x^{-2}) dx$
- $= 3y + \ln|y| + c$ $= \frac{3}{4}e^t + 2t^{\frac{3}{2}} + c$ $= \frac{1}{3}x^3 - 2x - x^{-1} + c$
- 4** $f'(x) = \frac{4x^2 - 4x + 1}{x} = 4x - 4 + \frac{1}{x}$
- $f(x) = \int \left(4x - 4 + \frac{1}{x}\right) dx = 2x^2 - 4x + \ln|x| + c$
- $(1, -3) \Rightarrow -3 = 2 - 4 + 0 + c$
- $\therefore c = -1$
- $f(x) = 2x^2 - 4x + \ln|x| - 1$
- 5** **a** $= [e^x + 10x]_0^1$ **b** $= \left[\frac{1}{2}t^2 + \ln|t|\right]_2^5$ **c** $= \int_1^4 \left(\frac{5}{x} - x\right) dx$
- $= (e + 10) - (1 + 0)$ $= \left(\frac{25}{2} + \ln 5\right) - (2 + \ln 2)$ $= [5\ln|x| - \frac{1}{2}x^2]_1^4$
- $= e + 9$ $= \frac{21}{2} + \ln \frac{5}{2}$ $= (5\ln 4 - 8) - (0 - \frac{1}{2})$
- $= 10\ln 2 - \frac{15}{2}$
- d** $= \int_{-2}^{-1} \left(2 + \frac{1}{3y}\right) dy$ **e** $= [e^x - \frac{1}{3}x^3]_{-3}^3$ **f** $= \int_2^3 (4 - 3r^{-1} + 6r^{-2}) dr$
- $= [2y + \frac{1}{3}\ln|y|]_{-2}^{-1}$ $= (e^3 - 9) - (e^{-3} + 9)$ $= [4r - 3\ln|r| - 6r^{-1}]_2^3$
- $= (-2 + 0) - (-4 + \frac{1}{3}\ln 2)$ $= e^3 - e^{-3} - 18$ $= (12 - 3\ln 3 - 2) - (8 - 3\ln 2 - 3)$
- $= 2 - \frac{1}{3}\ln 2$ $= 5 - 3\ln \frac{3}{2}$
- g** $= [7u - e^{4u}]_{\ln 2}^{\ln 4}$ **h** $= \int_6^{10} (2 + 9r^{-1}) dr$ **i** $= \int_4^9 (x^{-\frac{1}{2}} + 3e^x) dx$
- $= (7\ln 4 - 4) - (7\ln 2 - 2)$ $= [2r + 9\ln|r|]_6^{10}$ $= [2x^{\frac{1}{2}} + 3e^x]_4^9$
- $= 7\ln 2 - 2$ $= (20 + 9\ln 10) - (12 + 9\ln 6)$ $= (6 + 3e^9) - (4 + 3e^4)$
- $= 8 + 9\ln \frac{5}{3}$ $= 3e^9 - 3e^4 + 2$
- 6** $= \int_0^2 (3 + e^x) dx$ **7** $= \int_1^4 \left(2x + \frac{1}{x}\right) dx$
- $= [3x + e^x]_0^2$ $= [x^2 + \ln|x|]_1^4$
- $= (6 + e^2) - (0 + 1)$ $= (16 + \ln 4) - (1 + 0)$
- $= e^2 + 5$ $= 15 + 2\ln 2$

$$\begin{aligned} 8 \quad \mathbf{a} &= \int_0^1 (4x + 2e^x) \, dx \\ &= [2x^2 + 2e^x]_0^1 \\ &= (2 + 2e) - (0 + 2) = 2e \end{aligned}$$

$$\begin{aligned} \mathbf{c} &= \int_{-3}^{-1} (4 - \frac{1}{x}) \, dx \\ &= [4x - \ln|x|]_{-3}^{-1} \\ &= (-4 - 0) - (-12 - \ln 3) = 8 + \ln 3 \end{aligned}$$

$$\begin{aligned} \mathbf{e} &= \int_{\frac{1}{2}}^2 (e^x + \frac{5}{x}) \, dx \\ &= [e^x + 5 \ln|x|]_{\frac{1}{2}}^2 \\ &= (e^2 + 5 \ln 2) - (e^{\frac{1}{2}} + 5 \ln \frac{1}{2}) \\ &= e^2 - e^{\frac{1}{2}} + 10 \ln 2 \end{aligned}$$

$$\begin{aligned} 9 \quad \mathbf{a} \quad 9 - \frac{7}{x} - 2x &= 0 \\ 2x^2 - 9x + 7 &= 0 \\ (2x - 7)(x - 1) &= 0 \\ x &= 1, \frac{7}{2} \\ \therefore (1, 0) \text{ and } (\frac{7}{2}, 0) \end{aligned}$$

$$\begin{aligned} \mathbf{b} &= \int_1^{\frac{7}{2}} (9 - \frac{7}{x} - 2x) \, dx \\ &= [9x - 7 \ln|x| - x^2]_1^{\frac{7}{2}} \\ &= (\frac{63}{2} - 7 \ln \frac{7}{2} - \frac{49}{4}) - (9 - 0 - 1) \\ &= 11\frac{1}{4} - 7 \ln \frac{7}{2} \end{aligned}$$

$$\begin{aligned} 11 \quad \mathbf{a} \quad x = 3 \therefore y &= e^3 \\ \frac{dy}{dx} &= e^x \therefore \text{grad} = e^3 \\ \therefore y - e^3 &= e^3(x - 3) \quad [y = e^3(x - 2)] \end{aligned}$$

$$\mathbf{b} \text{ at } Q, y = 0 \therefore x = 2$$

$$\text{at } R, x = 0 \therefore y = -2e^3$$

$$\therefore Q(2, 0), R(0, -2e^3)$$

$$\begin{aligned} \mathbf{c} \text{ area under curve, } 0 \leq x \leq 3 \\ &= \int_0^3 e^x \, dx = [e^x]_0^3 = e^3 - 1 \end{aligned}$$

area of triangle under PQ

$$= \frac{1}{2} \times 1 \times e^3 = \frac{1}{2}e^3$$

area of triangle above QR

$$= \frac{1}{2} \times 2 \times 2e^3 = 2e^3$$

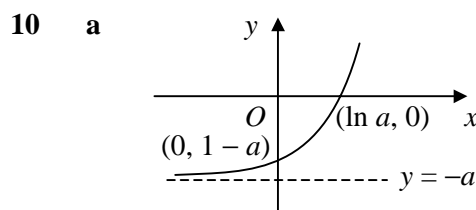
shaded area

$$= (e^3 - 1) - \frac{1}{2}e^3 + 2e^3 = \frac{5}{2}e^3 - 1$$

$$\begin{aligned} \mathbf{b} &= \int_2^4 (1 + \frac{3}{x}) \, dx \\ &= [x + 3 \ln|x|]_2^4 \\ &= (4 + 3 \ln 4) - (2 + 3 \ln 2) = 2 + 3 \ln 2 \end{aligned}$$

$$\begin{aligned} \mathbf{d} &= \int_0^{\ln 2} (2 - \frac{1}{2}e^x) \, dx \\ &= [2x - \frac{1}{2}e^x]_0^{\ln 2} \\ &= (2 \ln 2 - 1) - (0 - \frac{1}{2}) = 2 \ln 2 - \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \mathbf{f} &= \int_2^3 (x^2 - \frac{2}{x}) \, dx \\ &= [\frac{1}{3}x^3 - 2 \ln|x|]_2^3 \\ &= (9 - 2 \ln 3) - (\frac{8}{3} - 2 \ln 2) \\ &= \frac{19}{3} - 2 \ln \frac{3}{2} \end{aligned}$$



$$\begin{aligned} \mathbf{b} &= -\int_0^{\ln a} (e^x - a) \, dx = -[e^x - ax]_0^{\ln a} \\ &= -[(a - a \ln a) - (1 - 0)] = 1 - a + a \ln a \end{aligned}$$

$$\mathbf{c} \quad 1 - a + a \ln a = 1 + a$$

$$a \ln a = 2a, \ln a = 2, a = e^2$$

$$\begin{aligned} 12 \quad \mathbf{a} \quad (\frac{3}{\sqrt{x}} - 4)^2 &= 0 \\ \sqrt{x} &= \frac{3}{4} \end{aligned}$$

$$x = \frac{9}{16} \therefore (\frac{9}{16}, 0)$$

$$\begin{aligned} \mathbf{b} &= \int_{\frac{9}{16}}^1 (\frac{3}{\sqrt{x}} - 4)^2 \, dx \\ &= \int_{\frac{9}{16}}^1 (9x^{-1} - 24x^{-\frac{1}{2}} + 16) \, dx \\ &= [9 \ln|x| - 48x^{\frac{1}{2}} + 16x]_{\frac{9}{16}}^1 \\ &= (0 - 48 + 16) - (9 \ln \frac{9}{16} - 36 + 9) \\ &= -5 - 9 \ln \frac{9}{16} \approx 0.178 \end{aligned}$$