1.

i. Use algebraic division to express $\dfrac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}$ in the form $Ax + B + \dfrac{Cx + D}{x^2 - x - 6}$, where A, B, C and D are constants.

ii. Hence find $\displaystyle\int_4^6 \frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}\,dx$, giving your answer in the form $a + \ln b$.

[7]

2.

Express $\dfrac{2+x^2}{(1+2x)(1-x)^2}$ in partial fractions and hence show that $\displaystyle\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2}\,dx = \tfrac{1}{2}\ln\tfrac{3}{2} + \tfrac{1}{3}$.

[9]

3.

i. Express $\dfrac{x+8}{x(x+2)}$ in partial fractions.

[3]

ii. By first using division, express $\dfrac{7x^2+16x+16}{x(x+2)}$ in the form $P + \dfrac{Q}{x} + \dfrac{R}{x+2}$.

[3]

A curve has parametric equations $x = \dfrac{2t}{1-t},\ y = 3t + \dfrac{4}{t}$.

iii. Show that the cartesian equation of the curve is $y = \dfrac{7x^2+16x+16}{x(x+2)}$.

[4]

iv. Find the area of the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. Give your answer in the form $L + M\ln 2 + N\ln 3$.

[4]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	Clear start to algebraic division	M1	at least as far as x term in quot & subseq mult back	& attempt at subtraction
		i	(Quotient) = $x - 1$	A1		
		i	(Remainder) = $x + 7$	A1		
		i	Final answer: $x-1+\dfrac{x+7}{x^2-x-6}$	A1	final answer in correct form This must be shown in part (i) or, if not, then implied in part (ii) If no long division shown but only comparison of coefficients or otherwise, SR M0 B1 B1 B1 **Examiner's Comments** This question commenced with “Use algebraic division…..” and those candidates who did not follow this instruction were penalised. In general, the division was performed well and the positions of the quotient and remainder were rarely mixed up in the final expression.	Accept $A = 1, B = -1, C = 1, D = 7$
		ii	**Convert their** $\dfrac{Cx+D}{x^2-x-6}$ **to Partial Fracts**	M1		
		ii	$\dfrac{x+7}{x^2-x-6}=\dfrac{2}{x-3}-\dfrac{1}{x+2}$ <u>Their</u> …	A1A1	Correct fraction converted to correct PFs	
		ii	$\int Ax+B\,\mathrm{d}x=\dfrac{1}{2}Ax^2+Bx$ or $\dfrac{(Ax+B)^2}{2A}$	B1 ft		
		ii	$\int \dfrac{E}{x-3}+\dfrac{F}{x+2}\,\mathrm{d}x=E\ln(x-3)+F\ln(x+2)$	B1 ft		
		ii	Using limits in a correct manner	M1	Tolerate some wrong signs provided intention clear	

		ii	$8+\ln\frac{27}{4}$ $\left(8+\ln\frac{54}{8}\right)$ isw	A1	Answer required in the form $a + \ln b$, so giving only a decimalised form is awarded A0 Examiner's Comments The word "Hence" was used here and candidates were expected to use their expression from part (i) and evaluate the integral from that. The use of partial fractions was necessary but was applied by only a relatively small number of candidates.	
			Total	11		
2			$\frac{A}{1+2x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$ may be seen in later work	B1	or $\frac{A}{1+2x}+\frac{Bx+C}{(1-x)^2}$ may be seen later in later work	if B0M0, SC1 for $\frac{1}{1+2x}$ seen
			$2 + x^2 \equiv A(1 - x)^2 + B(1 + 2x)(1 - x) + C(1 + 2x)$	M1	or $A(1 - x)^2 + (Bx + C)(1 + 2x)$	allow only sign errors, not algebraic errors
			$A = 1$, $B = 0$ and $C = 1$ www	A1A1A1		
			$\int\left(\frac{1}{1+2x}+\frac{1}{(1-x)^2}\right)\mathrm{d}x=$ $a\ln(1 + 2x) + b(1 - x)^{-1}$	M1*	a and b are non-zero constants	ignore extra terms
			$\mathrm{F}(x) = ½\ln(1 + 2x) + (1 - x)^{-1}$	A1		
			their $\frac{1}{2}\ln(\frac{3}{2})+\frac{4}{3}-(\frac{1}{2}\ln 1+1)$	M1dep*		
			$\frac{1}{2}\ln(\frac{3}{2})+\frac{4}{3}-0-1$	A1	and completion to given result www Examiner's Comments Most recognised the correct form of partial fractions and successfully cleared the fractions. Although there were many fully correct solutions to this part of the questions, numerical slips such as $3C = 3$ so $C = 3$ and	NB $\frac{1}{2}\ln(\frac{3}{2})+\frac{1}{3}$

					$\frac{9}{4} = \frac{9}{4}A$ so $A = 9$ were surprisingly common. The integration was often well done, although $-(1-x)^{-1}$ was quite common, often leading to fudging of the subsequent arithmetic. As with 8(i), candidates are reminded of the need to show sufficient detail of the solution when working towards a given answer.	
			Total	9		
3		i	$\frac{A}{x}+\frac{B}{x+2}$	B1	allow one sign error **Examiner's Comments** Most recognised the correct form of partial fractions and successfully cleared the fractions to produce a fully correct solution.	award if only implied by answer
		i	$x + 8 = A(x + 2) + Bx$ soi	M1		clearing fractions successfully
		i	$A = 4$ *and* $B = -3$	A1		if **M0**, **B1** for each value www
		ii	quotient (P) is 7	B1	if **B0**, **B1** for $Q = 8$ and **B1** for $R = -6$ www **Examiner's Comments** Most candidates used long division and successfully found the quotient and the remainder. Many then used their answer to part (i) to produce a correct solution. A variety of other approaches were also successful, but a significant minority of those who equated coefficients went astray in the algebra. A small number of candidates tried to divide by x and x + 2	
		ii	$2x + 16$ seen	B1		eg as remainder or in division chunking

					separately, and were rarely successful.	
		ii	$7+\dfrac{8}{x}-\dfrac{6}{x+2}$	B1		or allow $P = 7$, $Q = 8$, $R = -6$
		iii	$t = \mathrm{f}(x)$	M1*	from $x=\dfrac{2t}{1-t}$ **M0** for $t = \mathrm{g}(y)$ or **B2** if unsupported <u>Examiner's Comments</u> There were many well laid out, perfectly correct responses to this question. However, it proved to be surprisingly difficult for many. Sometimes a formula for t in terms of x and t was substituted in, which didn't lead anywhere. In other cases the expression for t contained a sign error or an algebraic slip. Often candidates persisted with a clearly incorrect formula, instead of checking the early part of their work. A few candidates verified the result by substitution, which was a convoluted approach and did not earn full marks.	
		iii	$t=\dfrac{x}{x+2}$	A1		
		iii	$y = 3\times\text{their}\,\dfrac{x}{x+2}+\dfrac{4}{\text{their}\,\dfrac{x}{x+2}}$ eg $\dfrac{3x^2+(8+4x)(x+2)}{x(x+2)}$ and completion to	M1dep*		
		iii	$y=\dfrac{7x^2+16x+16}{x(x+2)}$	A1		at least one correct, constructive, intermediate step shown if **M0M0**, **SC2** for substitution of $x=\dfrac{2t}{1-t}$ n

						RHS of given equation and completion with at least two correct, constructive intermediate steps to $y = 3t + \frac{4}{t}$ www
		iv	$\int \text{their } (P + \frac{Q}{x} + \frac{R}{x+2})[\mathrm{d}x]$	M1*	where P, Q and R are constants obtained in (ii)	allow omission of dx
		iv	F[x] = $7x + 8\ln x - 6\ln(x + 2)$	A1ft	allow recovery from omission of brackets in subsequent working	if **M0**, **SC1** for $Px + Q\ln x + R\ln(x + 2)$ where constants are unspecified or arbitrary
		iv	F[2] – F[1]	M1dep*		
		iv	$7 - 4\ln 2 + 6\ln 3$	A1	<u>Examiner's Comments</u> There were many excellent responses to this part of the question. Most candidates spotted the link with part (ii) and went on to earn three or four marks. Those who started from scratch were almost never successful.	
			Total	14		