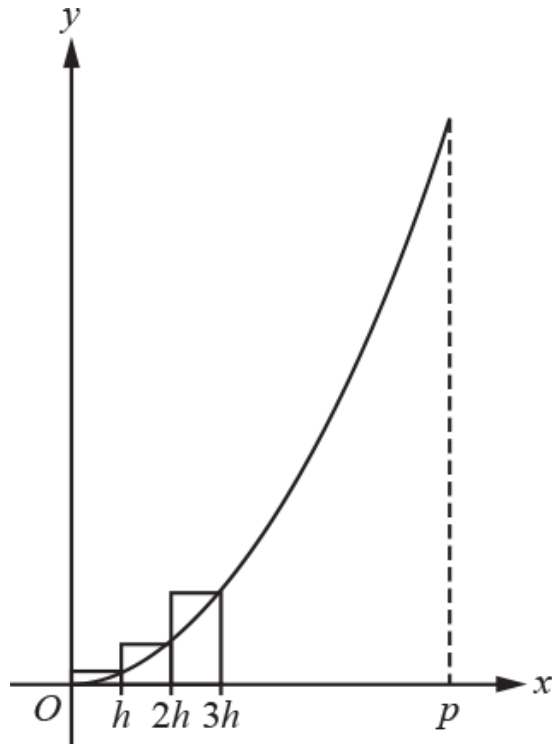


1. The diagram shows part of the curve  $y = x^2$  for  $0 \leq x \leq p$ , where  $p$  is a constant.



The area  $A$  of the region enclosed by the curve, the  $x$ -axis and the line  $x = p$  is given approximately by the sum  $S$  of the areas of  $n$  rectangles, each of width  $h$ , where  $h$  is small and  $nh = p$ . The first three such rectangles are shown in the diagram.

- (a) Find an expression for  $S$  in terms of  $n$  and  $h$ . [2]

- (b) Use the identity  $\sum_{r=1}^n r^2 \equiv \frac{1}{6}n(n+1)(2n+1)$  to show that  $S = \frac{1}{6}p(p+h)(2p+h)$ . [3]

- (c) Show how to use this result to find  $A$  in terms of  $p$ . [2]

END OF QUESTION paper

# Mark scheme

Question			Answer/Indicative content	Marks	Guidance							
1		a	<p>Heights are <math>h^2, (2h)^2, (3h)^2</math> etc</p> <p><math>S = h \times h^2 + h \times (2h)^2 + h \times (3h)^2 + \dots + h \times (nh)^2</math></p>	<p>B1 (AO1.1a)</p> <p>B1 (AO1.1)</p> <p>[2]</p>	<p>SOI</p> <p>or <math>h^3(1^2 + 2^2 + 3^2 + \dots + n^2)</math></p>	<p>or <math>h^3 = \sum_{r=1}^n r^2</math></p>						
		b	$S = h^3 \sum_{r=1}^n r^2$ $= \frac{h^3}{6} n(n+1)(2n+1)$ $= \frac{1}{6} nh(nh+h)(2nh+h)$ <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px;"><math>= \frac{1}{6} p(p+h)(2p+h)</math></td> <td style="padding: 2px;">AG</td> </tr> </table>	$= \frac{1}{6} p(p+h)(2p+h)$	AG	<p>M1 (AO3.1a)</p> <p>A1 (AO2.1)</p> <p>A1 (AO1.1)</p> <p>[3]</p>	<table border="1" style="width: 100%; height: 100px;"> <tr> <td style="width: 50%;"></td> <td style="width: 50%;"></td> </tr> <tr> <td style="text-align: center; vertical-align: middle;">oe</td> <td></td> </tr> </table>				oe	
$= \frac{1}{6} p(p+h)(2p+h)$	AG											
oe												
		c	$A = \lim_{h \rightarrow 0} S = \frac{1}{6} p \times p \times 2p$ $= \frac{p^3}{3}$	<p>M1 (AO2.5)</p> <p>A1 (AO2.2a)</p> <p>[2]</p>	<p>Correctly expressed limit statement</p> <p>Answer without working: M0A0</p>							
Total				7								