

1. (i) Given that $y = \ln\left(\frac{1 + \sin 4x}{\cos 4x}\right)$, show that $\frac{dy}{dx} = \frac{4}{\cos 4x}$. [4]

(ii) Find $\int \left(\frac{\cos 2x}{\cos 2x + \sin 2x} + \frac{\sin 2x}{\cos 2x - \sin 2x} \right) dx$. [4]

2. (a) Differentiate the following with respect to x .

(i) $\frac{1}{(3x - 4)^2}$ [1]

(ii) $\frac{\ln(x + 2)}{x}$ [3]

(b) Find $\int e^{(2x+3)} dx$. [3]

END OF QUESTION paper

Mark scheme

Question	Answer/Indicative content	Marks	Guidance
1 i	$\frac{\cos 4x \times 4 \cos 4x - (1 + \sin 4x) \times -4 \sin 4x}{\cos^2 4x}$ $\frac{4 \cos^2 4x + 4 \sin^2 4x + 4 \sin 4x}{\cos^2 4x} \text{ oe}$ $\frac{\cos 4x}{1 + \sin 4x} \times \text{their} \frac{4(1 + \sin 4x)}{\cos^2 4x}$ $= \frac{4}{\cos 4x} \text{ NB AG}$ <p><i>alternatively</i></p> $\frac{4 \cos 4x}{1 + \sin 4x} - \frac{-4 \sin 4x}{\cos 4x}$ $\frac{4 \cos 4x \times \cos 4x + 4 \sin 4x(1 + \sin 4x)}{(1 + \sin 4x) \cos 4x}$ <p>eg</p> $\frac{4(\cos^2 4x + \sin^2 4x) + 4 \sin 4x}{(1 + \sin 4x) \cos 4x}$ $\frac{4}{\cos 4x}$ <p><i>alternatively</i></p> $\frac{1}{\sec 4x + \tan 4x} \times (4 \sec 4x \tan 4x + 4 \sec^2 4x)$ $\frac{4 \sec 4x(\tan 4x + \sec 4x)}{\sec 4x + \tan 4x}$ <p>4sec4x</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>quotient rule; allow sign errors and / or one coefficient error</p> <p>use of chain rule; may be unsimplified</p> <p>chain rule; allow sign errors and / or one error in coefficient of $\cos 4x$ or $\sin 4x$</p> <p>combine to a single fraction FT <i>their</i> chain rule</p> <p>any equivalent correct step</p> <p>or use of product rule with $(1 + \sin 4x)$ and $(\cos 4x)^{-1}$ or $\sec 4x$</p> $(1 + \sin 4x) \times -1(\cos 4x)^{-2} \times -4 \sin 4x$ $+ \frac{4 \cos 4x}{\cos 4x}$

	$\frac{4}{\cos 4x}$	<p>[4]</p>	<table border="1"> <tr> <td data-bbox="1114 89 1321 689"> <p>allow sign errors and / or one coefficient error</p> <p>factorising – allow one coefficient slip</p> </td> <td data-bbox="1321 89 1551 689"></td> </tr> </table> <p>Examiner's Comments</p> <p>A variety of approaches were seen. Most went straight to the quotient rule and the chain rule and went on to derive the given result successfully. Almost as many separated the logarithms and used the chain rule before combining to a single fraction, and they were equally successful. A few converted to the reciprocal forms and were generally successful. A significant minority made slips with coefficients, signs and brackets, thus losing the accuracy marks. Candidates are reminded of the need for rigour when deriving a given result.</p>	<p>allow sign errors and / or one coefficient error</p> <p>factorising – allow one coefficient slip</p>	
<p>allow sign errors and / or one coefficient error</p> <p>factorising – allow one coefficient slip</p>					
<p>ii</p>	$\frac{\cos 2x(\cos 2x - \sin 2x) + \sin 2x(\cos 2x + \sin 2x)}{(\cos 2x + \sin 2x)(\cos 2x - \sin 2x)} \text{ oe}$ $\frac{\cos^2 2x - \cos 2x \sin 2x + \sin 2x \cos 2x + \sin^2 2x}{(\cos^2 2x - \sin^2 2x)}$ $\frac{1}{\cos 4x}$	<p>M1</p> <p>A1</p> <p>A1</p>	<table border="1"> <tr> <td data-bbox="1114 1496 1321 2096"> <p>combine into a single fraction; allow sign errors</p> <p>or better</p> </td> <td data-bbox="1321 1496 1551 2096"> <p>allow equivalent form with double angle formulae</p> <p>allow equivalent separate fractions with correct common denominator</p> </td> </tr> </table>	<p>combine into a single fraction; allow sign errors</p> <p>or better</p>	<p>allow equivalent form with double angle formulae</p> <p>allow equivalent separate fractions with correct common denominator</p>
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		$\frac{1}{4} \ln \left(\frac{1 + \sin 4x}{\cos 4x} \right) + c \text{ oe}$ $\text{eg } \frac{1}{4} \ln(1 + \sin 4x) + \frac{1}{4} \ln \sec 4x + c$	<p>A1</p> <p>[4]</p>	<table border="1"> <tr> <td style="text-align: center; vertical-align: middle;"> $\frac{1}{4}$ NB $\frac{1}{4}$ in (sec 4x + tan 4x) + c </td> <td></td> </tr> </table> <p>Examiner's Comments</p> <p>Candidates who failed to combine the integrand into a single fraction generally made no progress.</p> <p>Of the good number who did adopt the correct strategy, a significant proportion made sign or coefficient errors and so were unable to make the connection with part (i).</p>	$\frac{1}{4}$ NB $\frac{1}{4}$ in (sec 4x + tan 4x) + c							
$\frac{1}{4}$ NB $\frac{1}{4}$ in (sec 4x + tan 4x) + c												
	Total		8									
2	a	<table border="1" style="width: 100%;"> <tr> <td style="width: 5%; text-align: center;">(i)</td> <td style="width: 30%; text-align: center;"> $-\frac{6}{(3x-4)^3}$ </td> <td style="width: 65%;">or $-6(3x-4)^{-3}$ oe</td> </tr> </table> <table border="1" style="width: 100%;"> <tr> <td style="width: 5%; text-align: center;">(ii)</td> <td style="width: 30%; text-align: center;"> $\frac{x \times \frac{1}{x+2} - \ln(x+2)}{x^2}$ </td> <td style="width: 65%;"> $= \frac{1}{x(x+2)} - \frac{\ln(x+2)}{x^2}$ </td> </tr> </table>	(i)	$-\frac{6}{(3x-4)^3}$	or $-6(3x-4)^{-3}$ oe	(ii)	$\frac{x \times \frac{1}{x+2} - \ln(x+2)}{x^2}$	$= \frac{1}{x(x+2)} - \frac{\ln(x+2)}{x^2}$	<p>B1 (AO1.1) [1]</p> <p>M1 (AO1.1a) M1 (AO1.1)</p> <p>A1 (AO1.1) [2]</p>	<table border="1"> <tr> <td style="vertical-align: top;"> Allow M1 for denominator and one term in numerator correct oe two-layered fraction or fractions </td> <td></td> </tr> </table>	Allow M1 for denominator and one term in numerator correct oe two-layered fraction or fractions	
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	b	$\frac{1}{2} e^{(2x+3)} + c$	<p>B1 (AO1.1a) B1 (AO1.1) B1</p>	<table border="1"> <tr> <td style="vertical-align: top;"> B1 for $e^{(2x+3)}$ $\frac{1}{2}$ B1 for $\frac{1}{2}$ </td> <td></td> </tr> </table>	B1 for $e^{(2x+3)}$ $\frac{1}{2}$ B1 for $\frac{1}{2}$							
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				(AO1.1) [2]	B1 for + c
			Total	7	