1. The temperature of a freezer is –20°C. A container of a liquid is placed in the freezer. The rate at which the temperature, θ°C, of a liquid decreases is proportional to the difference in temperature between the liquid and its surroundings. The situation is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta + 20),$$

where time t is in minutes and k is a positive constant.

i. Express θ in terms of t, k and an arbitrary constant.

[3]

Initially the temperature of the liquid in the container is 40°C and, at this instant, the liquid is cooling at a rate of 3°C per minute. The liquid freezes at 0°C.

ii. Find the value of k and find also the time it takes (to the nearest minute) for the liquid to freeze.

[5]

The procedure is repeated on another occasion with a different liquid. The initial temperature of this liquid is 90°C. After 19 minutes its temperature is 0°C.

iii. Without any further calculation, explain what you can deduce about the value of k in this case.

[1]

2. At time t seconds, the radius of a spherical balloon is r cm. The balloon is being inflated so that the rate of increase of its radius is inversely proportional to the square root of its radius. When $t = 5$, $r = 9$ and, at this instant, the radius is increasing at 1.08 cm s^{-1}.

i. Write down a differential equation to model this situation, and solve it to express r in terms of t.

[7]

ii. How much air is in the balloon initially?

[2]

[The volume of a sphere is $V = \frac{4}{3}\pi r^3$.]

3. A container in the shape of an inverted cone of radius 3 metres and vertical height 4.5 metres is initially filled with liquid fertiliser. This fertiliser is released through a hole in the bottom of the container at a rate of 0.01 m^3 per second. At time *t* seconds the fertiliser remaining in the container forms an inverted cone of height *h* metres.

[The volume of a cone is $V=\frac{1}{3}\pi r^2 h$.]

i. Show that $h^2\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{9}{400\pi}$.

[5]

ii. Express *h* in terms of *t*.

[4]

iii. Find the time it takes to empty the container, giving your answer to the nearest minute.

[2]

4. In the year 2000 the population density, *P*, of a village was 100 people per km^2, and was increasing at the rate of 1 person per km^2 per year. The rate of increase of the population density is thought to be inversely proportional to the size of the population density. The time in years after the year 2000 is denoted by *t*.

i. Write down a differential equation to model this situation, and solve it to express *P* in terms of *t*.

[6]

ii. In 2008 the population density of the village was 108 people per km^2 and in 2013 it was 128 people per km^2. Determine how well the model fits these figures.

[2]

5.

i. Express $\frac{16+5x-2x^2}{(x+1)^2(x+4)}$ in partial fractions.

[5]

ii. It is given that

$$\frac{dy}{dx} = \frac{(16+5x-2x^2)y}{(x+1)^2(x+4)}$$

and that $y = \frac{1}{256}$ when $x = 0$. Find the exact value of y when $x = 2$. Give your answer in the form Ae^n.

[7]

6. Helga invests £4000 in a savings account. After t days, her investment is worth £y. The rate of increase of y is ky, where k is a constant.

(a) Write down a differential equation in terms of t, y and k. [1]

(b) Solve your differential equation to find the value of Helga's investment after t days. Give your answer in terms of k and t. [4]

It is given that $k = \frac{1}{365}\ln\left(1+\frac{r}{100}\right)$ where r% is the rate of interest per annum. During the first year the rate of interest is 6% per annum.

(c) Find the value of Helga's investment after 90 days. [2]

After one year (365 days), the rate of interest drops to 5% per annum.

(d) Find the total time that it will take for Helga's investment to double in value. [5]

7. A new bird species is introduced into a region where it has previously been absent. Initially 20 birds are introduced. The rate of increase of the number N of birds after t years is modelled by

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$$\frac{1}{1000}\,(10\,000 - N^2).$$

(a) Show that $N = \dfrac{100\,(3e^{0.2t} - 2)}{(3e^{0.2t} + 2)}$. [7]

(b) Hence explain what will happen to the number of birds over a long period of time, as predicted by the model. [2]

(c) State one limitation of the model. [1]

8. A tank is shaped as a cuboid. The base has dimensions 10 cm by 10 cm. Initially the tank is empty. Water flows into the tank at 25 cm^3 per second. Water also leaks out of the tank at $4h^2$ cm^3 per second, where h cm is the depth of the water after t seconds. Find the time taken for the water to reach a depth of 2 cm. [9]

9. A scientist is attempting to model the number of insects, N, present in a colony at time t weeks. When $t = 0$ there are 400 insects and when $t = 1$ there are 440 insects.

(a) A scientist assumes that the rate of increase of the number of insects is inversely proportional to the number of insects present at time t.

(i) Write down a differential equation to model this situation. [1]

(ii) Solve this differential equation to find N in terms of t. [4]

(b) In a revised model it is assumed that $\dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N^2}{3988\mathrm{e}^{0.2t}}$. Solve this differential equation to find N in terms of t. [6]

(c) Compare the long-term behaviour of the two models. [2]

10. The gradient of the curve $y = \mathrm{f}(x)$ is given by the differential equation

$$(2x-1)^3\frac{\mathrm{d}y}{\mathrm{d}x} + 4y^2 = 0$$

and the curve passes through the point (1, 1). By solving this differential equation show that

$$f(x) = \frac{ax^2 - ax + 1}{bx^2 - bx + 1} ,$$

where a and b are integers to be determined. [9]

11. The gradient function of a curve is given by $\frac{dy}{dx} = \frac{x^2 \sin 2x}{2\cos^2 4y - 1}$.

(a) Find an equation for the curve in the form $f(y) = g(x)$ [6]

The curve passes through the point $(\frac{1}{4}\pi, \frac{1}{12}\pi)$.

(b) Find the smallest positive value of y for which $x = 0$. [4]

12. As a spherical snowball melts its volume decreases. The rate of decrease of the volume of the snowball at any given time is modelled as being proportional to its volume at that time. Initially the volume of the snowball is 500 cm^3 and the rate of decrease of its volume is 20 cm^3 per hour.

(a) Find the time that this model would predict for the snowball's volume to decrease to 250 cm^3. [7]

(b) Write down one assumption made when using this model. [1]

(c) Comment on how realistic this model would be in the long term. [1]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	Separating variables $\int \frac{1}{\theta+20}\,\mathrm{d}\theta = \int -k\,\mathrm{d}t$	M1	or invert each side: $\frac{\mathrm{d}t}{\mathrm{d}\theta} = -\frac{1}{k(\theta+20)}$	Must see $\frac{1}{\theta+20}$; ignore posn 'k'
		i	$\ln(\theta + 20) = -kt$ (+ c) or equivalent	A1	"Eqn A" "Eqn B" **Examiner's Comments** The majority of candidates used the method of 'separating the variables' and generally integrated each side correctly. However, many did not go further or failed to rearrange this equation correctly in order to express θ in terms of *t, k* and an arbitrary constant. Many candidates thought that '+ c' should appear only at the end. A very few inverted each side and were generally less successful because of the position of '*k*' immediately alongside the θ + 20 in the denominator.	
		i	$\theta = Ae^{-kt} - 20$ oe (i.e. $0 = e^{-kt+c} - 20$)	A1		
		ii	$(-)3 = -k(40 + 20)$	M1	Using $t = 0$, $\theta = 40$, $\frac{\mathrm{d}\theta}{\mathrm{d}t} = (-)3$ in given equation Not $k = -\frac{1}{20}$	
		ii	$k = \frac{1}{20}$ oe	*A1	**Examiner's Comments** There were some very good and neat solutions to this part, particularly by those who read the question carefully before delving into its solution. Even though *k* was defined to be a	

					positive constant, a large number of candidates obtained $k = -\frac{1}{20}$. A few interpreted the statement "…at this instant, the liquid is cooling…." to imply that there was a constant decrease of 3 degrees every minute. However, many candidates attempted to use all the information correctly and found *k*, their own constant of integration and, finally, the time taken for the liquid to freeze.	
		ii	Subst $t = 0$, $\theta = 40$ & their *k* (where necessary) into their Eqn A or their Eqn B and solve for the arbitrary constant	M1		
		ii	Subst $\theta = 0$ & their values of k and the arbitrary constant into their Eqn A or their Eqn B	M1		
		ii	$t = 21.9722 = 22$ minutes cao www	dep* A1		
		iii	*k* is larger	B1	**Examiner's Comments** Most candidates stated that k must be larger, but explanations were sketchy.	
			Total	9		
2		i	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{\sqrt{r}}$ oe	B2	B1 for $\frac{\mathrm{d}r}{\mathrm{d}t} =$; B1 for $\frac{k}{\sqrt{r}}$	SR: B1 for $\frac{\mathrm{d}r}{\mathrm{d}t} \propto \frac{1}{\sqrt{r}}$
		i	Sep variables of their diff eqn (or invert) & integrate each side, increasing powers by 1 (or $\frac{1}{r} \to \ln r$)	*M1	their d.e. must be $\frac{\mathrm{d}r}{\mathrm{d}t}$ (or $\frac{\mathrm{d}t}{\mathrm{d}r}$) = f($r$)	Ignore absence of '+c' after integration

		i	Subst $\frac{dr}{dt} = 1.08, r = 9$ into their diff eqn to find k	M1	their d.e. must include $\frac{dr}{dt}$ (or $\frac{dt}{dr}$), r & k	(✔ k = 3.24 but M mark, not A)
		i	Substitute t = 5, r = 9 to find 'c'	dep*M1	Must involve '+c' here	
		i	Correct value of c (probably = 1.8 or –1.8)	A1	Check other values	
		i	$r = (4.86t + 2.7)^{\frac{2}{3}}$ SW	A1	Answer required in form r = f(t)	
		i			**Examiner's Comments** The section in the specification topic "First Order Differential Equations" requires candidates to formulate a simple statement involving a rate of change as a differential equation. This process is by no means understood by the majority of candidates — and particularly not in this case where inverse proportion was involved. Many of the candidates thought that the constant should be inverse, i.e. $\frac{1}{k}$, which would have worked through satisfactorily had they not thought that the inverse aspect was now fulfilled and they could just write $\frac{1}{k}\sqrt{r}$ The separation of variables and subsequent integration were performed reasonably. At the end, many candidates failed to express r in terms of t, as directed.	
		ii	subst t = 0 into any version of (i) result to find finite r	M1		(✔ r ≈ 1.938991….but M mark, not A)
		ii	Any V in range 30.5 ≤ V < 30.55, but not fortuitously	A1	Accept 9.72π or $\frac{243}{25}\pi$	

					Examiner's Comments Most understood what to do, gaining the method mark, and a few obtained the correct result.	
			Total	9		
3		i	$\frac{dV}{dt} = \pm 0.01$	B1		
		i	by similar triangles, $\frac{h}{4.5} = \frac{r}{3}$	B1	may be implied by $r = \frac{2h}{3}$ oe	
		i		B1	$\frac{dV}{dh} = \frac{4}{9}\pi h^2$ oe	
		i	$\frac{dh}{dt} = \pm 0.01 \times \text{their } \frac{dh}{dV}$ oe	M1	use of Chain rule	may follow from incorrect differentiation: expressions must be a function of either *r* or *h* or both
		i	$-0.01 = (\frac{4}{9}\pi h^2) \times \frac{dh}{dt}$ oe soi	A1	completion to given result www Examiner's Comments This defeated all but the best candidates. Many scored an easy B1 for $\frac{dV}{dt} = -0.01$ (although a good number missed out on this because they wrote $\frac{dh}{dt} = -0.01$ instead). Thereafter very few made any progress: the need to eliminate r was often not appreciated. Those who did spot the relationship between *h* and *r* often went on to score full marks. Having obtained *V* in terms of *h* only, a few strong	$h^2 \frac{dh}{dt} = \frac{-0.09}{4\pi} = \frac{-9}{400\pi}$

					candidates derived the required result by considering $V(t) = 13.5\pi - 0.01t$ and differentiating.	
		ii	$\int h^2 \mathrm{d}h = \int \frac{-9}{400\pi} \mathrm{d}t$	M1	separation of variables	if no subsequent work, integral signs needed, but allow omission of dh or dt, but must be correctly placed if present;
		ii	$\frac{h^3}{3} = \frac{-9}{400\pi} t (+c)$	A1		
		ii	substitution of $t = 0$ and $h = 4.5$ in their expression following integration	M1	expression must include c and powers must be correct on each side allow − 0.0215 or − 0.02148591…r.o.t to 4 sf or more and similarly 91.125	
		ii	$h = \sqrt[3]{\frac{729}{8} - \frac{27t}{400\pi}}$ oe isw	A1	**Examiner's Comments** This proved surprisingly difficult for many. Those who did separate the variables either differentiated h^2 instead of integrating, or omitted the constant of integration and made no further progress. Those who did achieve a correct value for "c" often went on to spoil their answer, or simply left the equation in implicit form.	$91.125 = {}^{729}/_{8}$
		iii	set $h = 0$ and solve to obtain positive t	M1	or (t =) $\frac{1}{3}\pi \times 3^2 \times 4.5 \div 0.01$ (= 1350π)	NB 1350π = 4241.150082…
		iii	71 minutes cao	A1	**Examiner's Comments** Most realised that setting $h = 0$ was required here. Due to earlier errors, this sometimes led to a negative value for t; surprisingly this did not always set the alarm bells ringing. A good proportion of candidates who did everything right lost an easy mark because they failed to convert the answer to	

					minutes. Some candidates who had made no progress earlier, were astute enough to realise that the correct answer could be obtained without the result from part (ii), as the rate of change of volume was constant.	Differential Equations
			Total	11		
4		i	$\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{k}{P}$	B1	or $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{1}{kP}$	*k* should be unspecified at this stage
		i	*k* = 100 from $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{k}{P}$	B1	or *k* = 0.01 from $\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{1}{kP}$	may be seen later
		i	$\int P\,\mathrm{d}P = \int$ (their *k*)$\mathrm{d}t$	M1*	allow *k* = 1	allow omission of ∫ and recovery of omission of one operator for **M1*A1**
		i	$\frac{P^2}{2}=kt+c$	A1	or $t=\frac{P^2}{2k}+d$	if **M0**, **SC2** for $\ln P = kt + c$ thereafter only **M1** may be earned
		i	substitution of $t = 0$ and $P = 100$	M1dep*	may follow incorrect algebraic manipulation, but equation must include *c* (or *d*)	NB $c = 5000$ or $d = -50$
		i	$P=\sqrt{10000+200t}$ or $10\sqrt{100+2t}$ or $P=\sqrt{200(50+t)}$ isw cao	A1		allow recovery from eg use of *x* for *P* throughout, but withhold final **A1** for eg $x=\sqrt{10000+200t}$
		i			<u>Examiner's Comments</u> A surprising number of candidates did not seem to understand inverse proportion, and setting up the initial equation elicited a wide range of incorrect responses. Those who did set up the equation correctly usually went on to earn at least four marks out of six. Finding k caused more difficulty than expected: many candidates mistakenly assuming that *t* = 1, *P* = 101 was a valid pair of values, instead of working with the information given in the question. In some cases	

					rearranging to make P the subject of the formula proved troublesome.	Differential Equations
		ii	$t = 8$, $P = 107.7$ or 108 so model was a good fit in 2008 oe	B1	or $t = 8.3(2)$ when $P = 108$ + comment	value of P or t must be found and correct comment made in each case; comments may be in same sentence.
		ii	$t = 13$, $P = 112(.2)$, so model was not appropriate in 2013 oe	B1	or $t = 31.9(2)$ or 32 when $P = 128$ + comment comments may be in same sentence, but both values must be referenced	if **B0B0**, **SC1** for both values found no FT marks available comments on trends, extrapolation etc do not score just ticks / crosses etc do not score
		ii			<u>Examiner's Comments</u> Full marks were rarely achieved in this case. Those who did find the correct values often speculated on future trends rather than commenting on the two values they had found. It was necessary to have earned at least four marks in part (i) to score in this part.	
			Total	8		
5		i	$\dfrac{A}{(x+4)}+\dfrac{B}{(x+1)}+\dfrac{C}{(x+1)^2}$	B1		may be awarded later
		i	$[16 + 5x - 2x^2] = A(x + 1)^2 + B(x + 1)(x + 4) + C(x + 4)$	M1		allow sign errors only
		i	$A = -4$	A1	**NB** $36 = -9A$	if **B0M0**, allow **SC3** for $\dfrac{2x+5}{(x+1)^2}-\dfrac{4}{x+4}$
		i	$C = 3$	A1	$9 = 3C$	

		i	$B = 2$ isw	A1	$-2 = A + B$, $5 = 2A + 5B + C$ $16 = A + 4B + 4C$ **NB** $\frac{-4}{(x+4)} + \frac{2}{(x+1)} + \frac{3}{(x+1)^2}$	**Examiner's Comments** Most recognised the correct form of partial fractions and successfully cleared the fractions to produce a fully correct solution.
		ii	$\int \frac{\mathrm{d}y}{y} = \int \frac{16 + 5x - 2x^2}{(x+1)^2(x+4)} \mathrm{d}x$	B1	separation of variables	allow omission of integral signs; allow omission of dy or dx but not both
		ii	$\frac{3}{(x+1)^2} + \frac{2}{(x+1)} - \frac{4}{(x+4)}$ seen in RHS, may be embedded	M1*	**FT** their partial fractions if two or three terms; ignore LHS	may be implied by correct integration of two of their terms
		ii	$\frac{-3}{x+1} + 2\ln(x+1) - 4\ln(x+4) + c$	A1FT	**FT** their non-zero 3, 2 and 4; allow recovery from $x + 1^2$ in denominator; if brackets in log terms omitted, allow **A1** if recovery seen in substitution	allow omission of + c here
		ii	$\ln(\frac{1}{256}) = -3 + 2\ln 1 - 4\ln 4 + c$	M1*dep	substitution of $x = 0$ and $y = \frac{1}{256}$	+ c must be included and LHS must be correctly obtained
		ii	$c = 3$ cao	A1	allow if error in manipulation following integration; or $A = \mathrm{e}^{-3}$ from $y = Ae^{\frac{-3}{x+1}} \frac{(x+1)^2}{(x+4)^4}$	
		ii	$\ln y = \frac{-3}{2+1} + 2\ln(2+1) - 4\ln(2+4) + 3$	M1*dep	substitution of $x = 2$; dependent on award of previous **M1M1** and numerical value found	allow **M1** if substitution follows incorrect manipulation

		ii	$y = \dfrac{e^2}{144}$ oe	A1	for c	eg to find expression for y **Examiner's Comments** The separation of variables caused problems for many, but most recognised the link with part (i) and worked with their partial fractions. The method was often well understood, but only a minority of candidates had the stamina and attention to detail to go on to the end and achieve the correct result.
			Total	12		
6		a	$\dfrac{\mathrm{d}y}{\mathrm{d}t} = ky$	B1(AO3.1b) [1]		
		b	$\dfrac{\mathrm{d}y}{y} = k\mathrm{d}t$ $\left[\ln y\right]_{4000}^{y} = k\left[t\right]_0^t$ or $\ln y = kt + c$ $\ln \dfrac{y}{4000} = kt$ or $\ln 4000 = 0 + c$ $y = 4000e^{kt}$	M1(AO1.1a) M1(AO1.1) A1(AO1.1) A1(AO1.1) [4]	Attempt separation of variables Correct integrals and limits Correct substitution in correct integral	
		c	$4000e^{\frac{90}{365}\ln 1.06}$ = 4057.89	M1(AO1.1) A1(AO1.1) [2]	FT their part (b) BC	

		d	After 1 year, increased by factor 1.06 Require further increase by factor $\frac{2}{1.06}$ $e^{\frac{t}{365}\ln 1.05} = \frac{2}{1.06}$ $\frac{t}{365}\ln 1.05 = \ln\frac{2}{1.06}$ $t = \frac{365}{\ln 1.05} \times \ln\frac{2}{1.06}$ $= 4750$ Total number of days = 5115	M1(AO3.1b) M1(AO1.1) A1(AO2.1) M1(AO1.1) A1(AO3.2a) [5]	May be implied Attempt to form equation with 1.05 and 1.06 Correct equation Attempt to remove logs isw	OR BC
			Total	12		
7		a	$\frac{dN}{dt} = \frac{1}{1000}(10000 - N^2)$ $\frac{dN}{10000 - N^2} = \frac{1}{1000}dt$ $\frac{1}{200}\left(\frac{1}{100-N} + \frac{1}{100+N}\right)dN = \frac{1}{1000}t$ $\ln\frac{100+N}{100-N} = 0.2t + c$	M1(AO 3.3) M1(AO 1.1) M1(AO 3.1b) A1(AO 2.1) M1(AO 1.1)	Attempt separate variables Attempt PFs with correct denominators Equation of form	

			when $t = 0$, $N = 20$ hence $c = \ln(3/2)$ $\ln\frac{2(100+N)}{3(100-N)} = 0.2t$ $\frac{2(100+N)}{3(100-N)} = e^{0.2t}$ $N = \frac{100(3e^{0.2t}-2)}{(3e^{0.2t}+2)}$ **AG**	A1(AO 2.1) A1(AO 1.1) [7]	$k\ln\frac{100+N}{100-N} = k't \quad (+c)$ Attempt find c with $t = 0$ and $N = 20$ Any correct equation in N & t after antilog Correctly rearrange to give answer	Allow without $+c$
		b	No. of birds increases and tends to 100	E1(AO 2.2b) E1(AO 3.4) [2]		
		c	Eg N is discrete but modelled by continuous	E1(AO 3.5b) [1]	No account of external factors eg weather	
			Total	10		
8			$V = 100h \Rightarrow \frac{dv}{dh} = 100$ $\frac{dv}{dt} = \frac{dv}{dh}\times\frac{dh}{dt} = 100\frac{dh}{dt} \qquad [= 25 - 4h^2]$	M1(AO 3.4) A1(AO 1.2) M1(AO 3.1b)		

			$\Rightarrow 25 - 4h^2 = 100\dfrac{dh}{dt}$ oe	M1(AO 2.5)	Equate $25 - 4h^2$ to their $\dfrac{dv}{dh} \times \dfrac{dh}{dt}$		
			$\Rightarrow \int_0^2 \frac{1}{25-4h^2}\,dh = \int_0^t \frac{1}{100}\,dt$	M1(AO 3.4)	Attempt integration with correct denominator on LHS		
			$\Rightarrow \frac{1}{10}\int_0^2 \frac{1}{5+2h} + \frac{1}{5-2h}\,dh = \int_0^t \frac{1}{100}\,dt$	A1(AO 2.1)			
				M1(AO 1.2)	Attempt partial fractions with correct denominators on LHS		
				A1(AO 2.2a)	Correct partial fractions		
			$\Rightarrow \dfrac{1}{10} \times \dfrac{1}{2}\left[\ln(5+2h) - \ln(5-2h)\right]_0^2 = \dfrac{t}{100}$	A1(AO 3.2a)	Correct integral; ignore limits		
			$\Rightarrow 5\ln 9 = t$ oe	[9]	Any correct numerical expression for t	10.9861…	
			Time when depth is 2 cm is 11.0 seconds (3 sf)		Allow 11 seconds		

			Total	9		
9		a	(i) $\frac{dN}{dt} = \frac{k}{N}$	B1 (AO 3.3) [1]	State correct differential equation | Or $\frac{dN}{dt} = \frac{1}{k'N}$ or equiv with k on LHS **Examiner's Comments** The majority of candidates could state a correct differential equation, including a coefficient of proportionality. The most common error was to have the equation as a function of t not of N.	
		a	(ii) $kt = \int N\mathrm{d}N$ $kt = \frac{1}{2}N^2 + c$ $0 = 80000 + c \Rightarrow c = -80000$ $k = 96800 - 80000 = 16800$	M1* (AO 2.1) M1d* (AO 3.4) M1d* (AO 3.4)	Attempt integration | Obtain equation of form $at = bN^2 + c$ Condone no $+ c$ Attempt c from (0, 400) | Substitute (0, 400) into their equation containing c and k Could give value for c, or could result in an equation involving both c and k	

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	$N = \sqrt{(33600t + 160000)}$	A1 (AO 1.1) [4]	

Attempt k from (1, 440)	depending on structure Substitute (1, 440) into their equation containing c (possibly now numerical) and k If c is numerical then value of k must be attempted If this gives second equation in c and k then the equations need to be solved simultaneously for c and k to award M1
Correct equation for N	N must be the subject of the equation

Examiner's Comments

Candidates who had stated a correct equation in part **(a)** were usually able to rearrange and integrate correctly. They were then able to use the given boundary conditions to find a particular solution to the differential equation. Having got this

					far, a few candidates did not gain the final mark due to making errors when rearranging the equation to the requested form, such as square rooting term by term.	
	b		$\int 3988N^{-2}\,dN = \int e^{-0.2t}\,dt$ $-3988N^{-1} = -5e^{-0.2t} + c$ OR $-N^{-1} = -\frac{5}{3988}e^{-0.2t} + c$ $-9.97 = -5 + c \Rightarrow c = -4.97$ OR $-\frac{1}{400} = -\frac{5}{3988} + c \Rightarrow c = -\frac{497}{398800}$	M1 (AO 3.1a) M1* (AO 1.1a) A1 (AO 1.1) M1d* (AO 2.2a)	<table><tr><td>Separate variables and attempt integration Integrate to obtain answer of correct form Obtain correct integral Attempt c from (0, 400) or (1, 440)</td><td>Must be valid method to separate variables so allow coefficient slips only Some attempt to integrate, but may not be correct BOD if no integral signs, as long as integration is actually attempted Obtain integral of the form $aN^{-1} = be^{-0.2t} + c$ or equiv Condone no + c Any equivalent form As far as attempting numerical value for c</td></tr></table>	

			$\frac{3988}{N} = 5e^{-0.2t} + 4.97$ OR $\frac{1}{N} = \frac{5}{3988}e^{-0.2t} + \frac{497}{398800}$	M1d* (AO 1.1)	Attempt to make N the subject	NB (0, 400) gives –4.97, (1, 440) gives an answer which rounds to –4.97 Equation may no longer be correct Using correct algebraic processes throughout, but allow sign slips – this includes any rearrangement attempt made prior to attempting c Must involve a c, either numerical or still as c
			$N = \frac{3988}{5e^{-0.2t}+4.97}$	A1 (AO 1.1) [6]	Correct equation for N	Any correct equation of form $N = \ldots$

					Examiner's Comments Candidates identified the need to separate the variables before integrating and many made a good attempt to do so. The majority of candidates chose to leave the 3988 where it was rather than incorporating it with the term in N. This caused problems for some when rewriting the right-hand side in a form that could be integrated, with $3988e^{-0.2t}$ being the most common error. When finding a value for the constant of integration most candidates opted to use the first condition given. Once again, not all candidates were able to use the correct method to find N in terms of t; the most common error was to simply invert each of the three terms in their equation.	
		c	Model in (a) predicts that population will continue to increase Model in (b) predicts that population will tend towards a limit of 802	E1 (AO 3.5a) E1 (AO 3.5a)	Comment about continuing to increase — Allow comments such as tending to infinity. Any additional comments must also be correct so E0 for eg 'will always increase at a steady rate' but E1 for 'will always increase but the rate of increase will decrease' Comment about tending towards a limit of 802 — Allow a limit of 803 or 802.4. Must come from a fully correct function	

				[2]	Any additional comments must also be correct **Examiner's Comments** When considering the first model, candidates simply needed to identify that it predicted that the population would continue to increase and a number of candidates were able to do so. Some spoilt an otherwise correct statement by including additional, incorrect detail such as increasing at a faster rate. Only the most able candidates were able to both identify that the second model would predict that the population will tend towards a limit and also identify the value of the limit. There was no credit for commenting on how realistic each of the two models was, but some candidates did consider this.	
			Total	13		
10			$(2x-1)^3\frac{\mathrm{d}y}{\mathrm{d}x}+4y^2=0$ $-\frac{1}{4}\int\frac{\mathrm{d}y}{y^2}=\int\frac{\mathrm{d}x}{(2x-1)^3}$ $\int\frac{\mathrm{d}y}{y^2}=-\frac{1}{y}$ $\int\frac{\mathrm{d}x}{(2x-1)^3}=\frac{(2x-1)^{-2}}{(2)(-2)}$	M1(AO 2.5)E A1(AO 1.1)E M1(AO 1.1)E A1(AO 1.1)C	Attempt to separate variables M1 for $k(2x-1)^{-2}$	

			Answer	Marks	Guidance		
			$\frac{1}{4y} = -\frac{1}{4(2x-1)^2} + c$, $(1,1) \Rightarrow c = \ldots$	M1(AO 2.1)C	Use of (1, 1) to find c – dependent on the previous two M marks and substituted into correct form		
			$\frac{1}{y} = -\frac{1}{(2x-1)^2} + 2$	A1(AO 2.2a)A			
			$\frac{1}{y} = \frac{2(2x-1)^2 - 1}{(2x-1)^2}$	M1(AO 3.1a)A	Oe	Or re–write in terms of y	
			$y = \frac{(2x-1)^2}{2(2x-1)^2 - 1}$	M1(AO 1.1)A	Correct method for combining both terms on rhs (dependent on previous M mark) before taking the reciprocal	Remove tripledecker fractions	
			$y = \frac{4x^2 - 4x + 1}{8x^2 - 8x + 1}$	A1(AO 2.2a)A [9]	Taking the reciprocal (dependent on previous M marks) and making y the subject $a = 4$, $b = 8$		

					Examiner's Comments The responses to this final question in the pure section were mixed with examiners reporting a mixture of excellent responses followed by those that struggled with both the integration and the resulting algebraic manipulation required to obtain the answer in the required form. While most correctly separated the variables and wrote $-\frac{1}{4}\int\frac{dy}{y^2}=\int\frac{dx}{(2x-1)^3}$ many candidates had issues with the placement of the fraction on the left–hand side with examiners reporting that frequently this became a 4 rather than remaining as a quarter. While many candidates correctly integrated and remembered to include an arbitrary constant many decided to re–arrange their equation before attempting to find this constant; candidates are advised that in the majority of situations it is probably wisest to work out the $+c$ immediately. Of those that obtained a correct particular solution to this differential equation, for example, $\frac{1}{y}=-\frac{1}{(2x-1)^2}+2$ many did not know the correct method for obtaining the result for $f(x)$ in the required form. Many candidates took the reciprocal of each term separately rather than combining all relevant fractions first before taking the reciprocal and then expanding the brackets.	
			Total	9		
11		a	$\int(2\cos^2 4y-1)dy=\int x^2\sin 2x\,dx$ $2\cos^2 4y-1=\cos 8y$	M1 (AO 1.1a) M1 (AO	Separate variables	

				3.1a)		
			$\int \cos 8y\,dy = \frac{1}{8}\sin 8y + c_1$	A1 (AO 1.1)	Attempt use of double angle formula	Obtain $\pm\cos 8y$
			$\int x^2 \sin 2x\,dx = -\frac{1}{2}x^2\cos 2x + \int x\cos 2x\,dx$	M1 (AO 3.1a)	Obtain correct integral	Condone no + c_1
			$= -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x\,dx$	M1 (AO 1.1)	Attempt integration by parts once	
			$= -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c_2$	A1 (AO 1.1)	Attempt second integration by parts	
			$\frac{1}{8}\sin 8y = -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	[6]	Obtain correct integral	Condone no + c_2
		b	$\text{Sin}\frac{8}{12}\pi = -4\left(\frac{1}{4}\pi\right)^2\cos\frac{\pi}{2} + 4\left(\frac{1}{4}\pi\right)\sin\frac{\pi}{2} + 2\cos\frac{\pi}{2} + c.$	M1 (AO 1.1)	Attempt c, using $x = \frac{1}{4}\pi$, $y = \frac{1}{12}\pi$	
			$c = \frac{1}{2}\sqrt{3} - \pi$	A1 (AO 1.1)	Obtain	

			Sin $8y=2+\frac{1}{2}\sqrt{3}-\pi$ $8y$ = (–0.279), 3.421 y = (–0.035), 0.428 y = 0.428	M1 (AO 3.1a) A1 (AO 1.1) [4]	correct value for c for their correct equation Condone decimal equiv (–2.276) Attempt positive value for y when $x = 0$ Obtain correct value for y	eg $c=\pi-\frac{1}{2}\sqrt{3}$ if their c on LHS, or $c=\pm\left(\frac{1}{16}\sqrt{3}-\frac{1}{8}\pi\right)$ if fractions not yet cleared A0 if extra values
			Total	10		
12		a	$\frac{dV}{dt}=-kV$ $-20 = -k \times 500$ so $k = 0.04$	B1(AO 3.3) B1(AO 3.3) M1(AO 1.1)	Set up correct differential equation Correct value for k – may be seen later	Allow k for $-k$ Or $k = -0.04$

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			Answer	Marks	Guidance	
			$\int -0.04\mathrm{d}t = \int \frac{1}{V}\mathrm{d}V$	A1(AO 1.1)	Separate variables and attempt integration	
			$-0.04t = \ln V + c$	M1* (AO 3.4)	Correct integral – could still be in terms of k	Accept no + c here
			$c = -\ln 500$	M1dep*(AO 3.4)	Use $t = 0$, $V = 500$ to find c	
			$-0.04t = \ln 250 - \ln 500$	A1 (AO 3.3)	Attempt to find t when $V = 250$	
			$t = 17.3$ hours	[7]	Obtain 17.3 hours, or better (17 hours and 20 minutes)	Units needed 17.3286…
			Alternate method	B1(AO 3.3)		
			$\frac{\mathrm{d}V}{\mathrm{d}t} = -kV$	B1(AO 3.3)		
			$-20 = -k \times 500$ so $k = 0.04$	M1(AO 1.1a)	Set up correct differential equation	Allow k for $-k$
			$\int_0^T -0.04\mathrm{d}t = \int_{500}^{250} \frac{1}{V}\mathrm{d}V$	M1(AO 3.4)	Correct value for k – may be seen later	Or $k = -0.04$
				M1(AO 3.4)	Separate variables and attempt integration of LHS	
				A1(AO 1.1)	Use of $t = 0$, $V = 500$	**OR** Use of t limits 0 and T (accept $t =$
				A1(AO 3.4)		
				[7]		

			$-0.04T = -0.693\ldots$ $T = 17$ hours		t) Use of $t = T$, $V = 250$ (accept $t = t$) Obtain 17.3 hours, or better (17 hours and 20 minutes) Use of V limits 500 and 250 (either way round) Units needed 17.3286…	
		b	E.g. Assumes that temperature remains constant E.g. Assume that the snowball remains a sphere throughout	B1(AO 3.5b) [1]	Any valid assumption made	
		c	Not very realistic as volume never equals 0, so snowball never melts completely	B1(AO 3.5b) [1]	Consider long term prediction	
			Total	9		

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