1. Fig. 8 shows parts of the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, where $\mathrm{f}(x) = \tan x$ and $\mathrm{g}(x) = 1 + \mathrm{f}(x - \frac{1}{4}\pi)$.

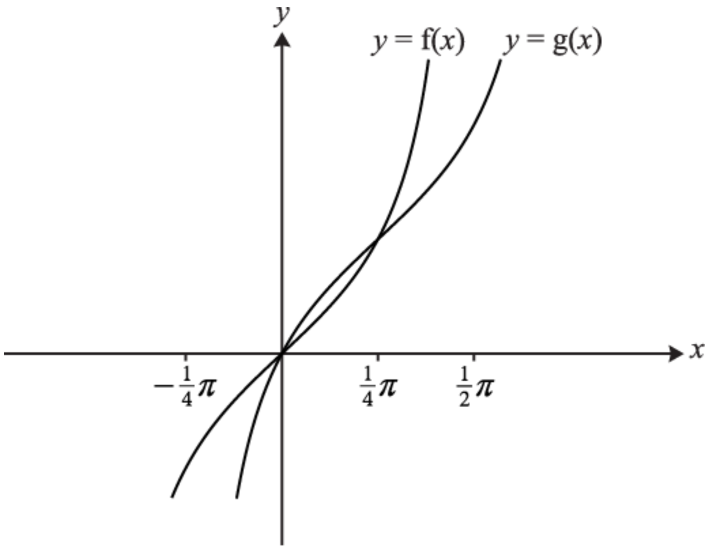


Fig. 8

i. Describe a sequence of two transformations which maps the curve $y = \mathrm{f}(x)$ to the curve $y = \mathrm{g}(x)$.

[4]

It can be shown that $\mathrm{g}(x) = \dfrac{2\sin x}{\sin x + \cos x}$.

ii. Show that $\mathrm{g}'(x) = \dfrac{2}{(\sin x + \cos x)^2}$. Hence verify that the gradient of $y = \mathrm{g}(x)$ at the point $(\frac{1}{4}\pi, 1)$ is the same as that of $y = \mathrm{f}(x)$ at the origin.

[7]

iii. By writing $\tan x = \dfrac{\sin x}{\cos x}$ and using the substitution $u = \cos x$, show that $\displaystyle\int_0^{\frac{1}{4}\pi} \mathrm{f}(x)\,\mathrm{d}x = \int_{\frac{1}{\sqrt{2}}}^{1} \frac{1}{u}\,\mathrm{d}u$.

Evaluate this integral exactly.

[4]

iv. Hence find the exact area of the region enclosed by the curve $y = \mathrm{g}(x)$, the x-axis and the lines $x = \frac{1}{4}\pi$ and $x = \frac{1}{2}\pi$.

[2]

2. Evaluate $\int_0^3 x(x+1)^{-\frac{1}{2}}\,dx$, giving your answer as an exact fraction.

[5]

3. Fig. 9 shows the curve with equation $y^3 = \dfrac{x^3}{2x-1}$. It has an asymptote $x = a$ and turning point P.

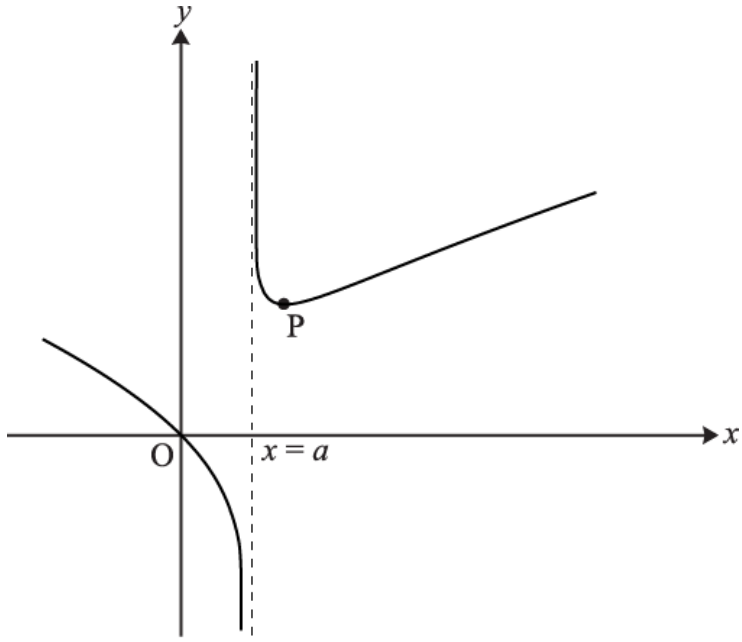


Fig. 9

i. Write down the value of a.

[1]

ii. Show that $\dfrac{dy}{dx} = \dfrac{4x^3 - 3x^2}{3y^2(2x-1)^2}$.

Hence find the coordinates of the turning point P, giving the y-coordinate to 3 significant figures.

[9]

iii. Show that the substitution $u = 2x - 1$ transforms $\int \dfrac{x}{\sqrt[3]{2x-1}}\,dx$ to $\dfrac{1}{4}\int \left(u^{\frac{2}{3}} + u^{-\frac{1}{3}}\right)du$.

Hence find the exact area of the region enclosed by the curve $y^3 = \dfrac{x^3}{2x-1}$, the x-axis and the lines $x = 1$ and $x = 4.5$.

[8]

4.

Using a suitable substitution or otherwise, show that $\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{3+\cos 2x}\,\mathrm{d}x = \frac{1}{2}\ln 2$.

[5]

5.

Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{x}{\sqrt{2+x^2}}$.

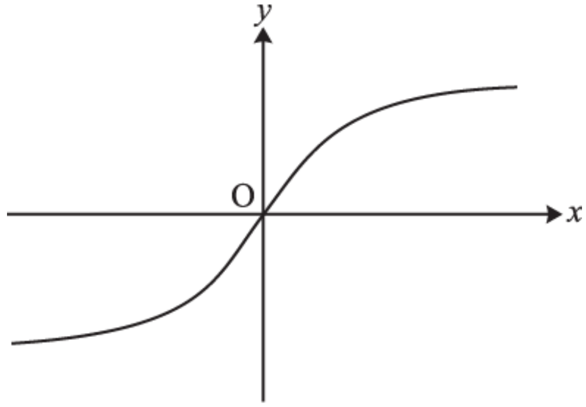


Fig. 8

i. Show algebraically that f (x) is an odd function. Interpret this result geometrically.

[3]

ii. Show that $\mathrm{f}'(x) = \frac{2}{(2+x^2)^{\frac{3}{2}}}$. Hence find the exact gradient of the curve at the origin.

[5]

iii. Find the exact area of the region bounded by the curve, the x-axis and the line $x = 1$.

[4]

iv.

A. Show that if $y = \frac{x}{\sqrt{2+x^2}}$, then $\frac{1}{y^2} = \frac{2}{x^2} + 1$.

[2]

B. Differentiate $\frac{1}{y^2} = \frac{2}{x^2} + 1$ implicitly to show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y^3}{x^3}$. Explain why this expression cannot be used to

find the gradient of the curve at the origin.

[4]

6. Fig. 9 shows the curve $y = \text{f}(x)$, where

$$\text{f}(x) = (\text{e}^x - 2)^2 - 1, \ x \in \mathbb{R}.$$

The curve crosses the x-axis at O and P, and has a turning point at Q.

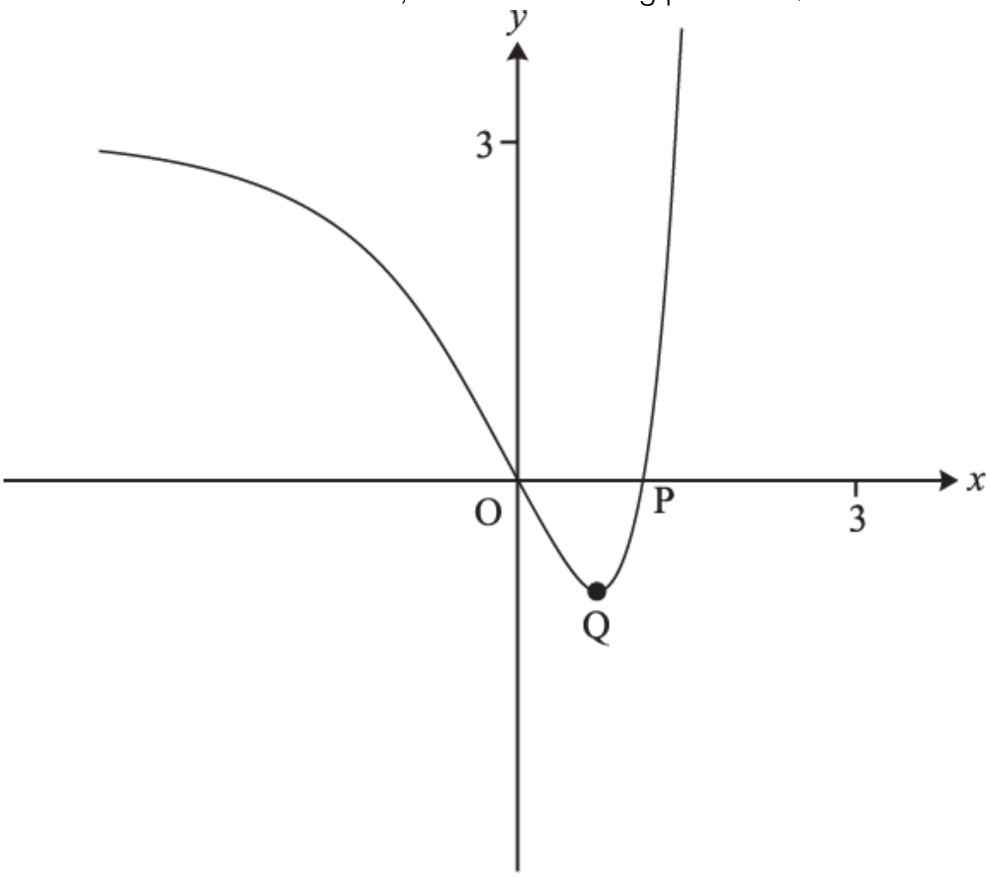


Fig. 9

i. Find the exact x-coordinate of P.

[2]

ii. Show that the x-coordinate of Q is ln 2 and find its y-coordinate.

[4]

iii. Find the exact area of the region enclosed by the curve and the x-axis.

[5]

The domain of $\text{f}(x)$ is now restricted to $x \geq \ln 2$.

iv. Find the inverse function $\text{f}^{-1}(x)$. Write down its domain and range, and sketch its graph on the copy of Fig. 9.

[7]

7. Find $\int \sqrt[3]{2x-1}\,dx$. [4]

8. Fig. 8 shows the curve $y = \dfrac{x}{\sqrt{x+4}}$ and the line $x = 5$. The curve has an asymptote l.

The tangent to the curve at the origin O crosses the line l at P and the line $x = 5$ at Q.

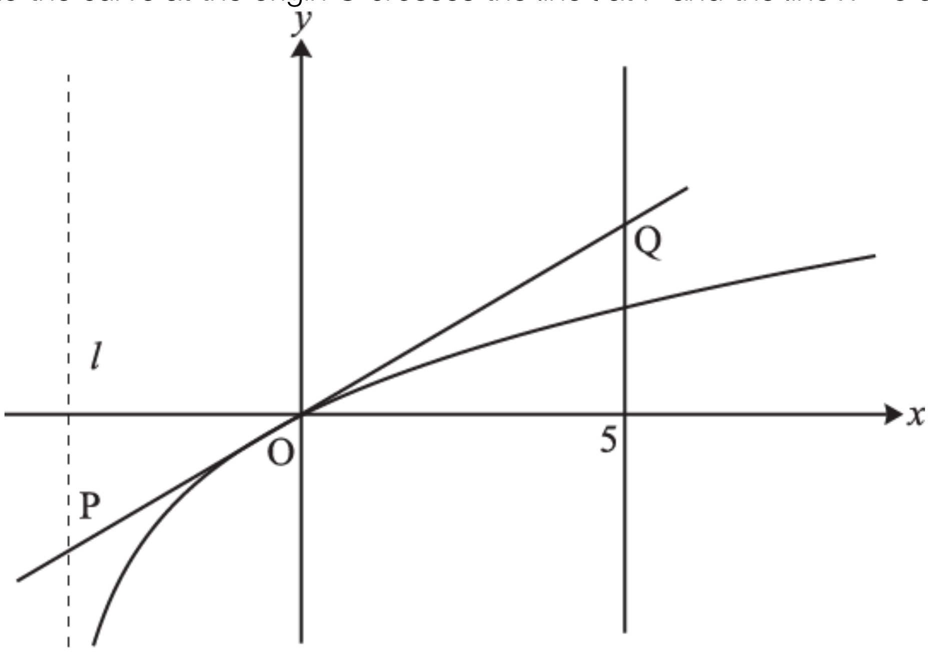


Fig. 8

i. Show that for this curve $\dfrac{dy}{dx} = \dfrac{x+8}{2(x+4)^{\frac{3}{2}}}$. [5]

ii. Find the coordinates of the point P. [4]

iii. Using integration by substitution, find the exact area of the region enclosed by the curve, the tangent OQ and the line $x = 5$. [9]

9. Evaluate $\int_0^1 \dfrac{1}{1+\sqrt{x}}\,dx$, giving your answer in the form $a + b \ln c$, where a, b and c are integers. [6]

10. Find $\int 18x(3x+1)^7 \mathrm{d}x$.

You may wish to use the substitution $u = (3x + 1)$. [6]

11. A curve has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^3 e^{-x^2}$.

(i) Show that $\mathrm{f}(x)$ is an odd function, and interpret this result in terms of the graph of the curve $y = \mathrm{f}(x)$. [3]

(ii) Find the coordinates of the stationary points of the curve. Give answers correct to 2 decimal places where appropriate. [7]

(iii) Sketch the curve for $-2 \le x \le 2$. [2]

(iv) (*A*) Show, using the substitution $t = x^2$, that $\int \mathrm{f}(x)\mathrm{d}x$ may be expressed as $\int kt\, e^{-t}\, \mathrm{d}t$, where k is a constant to be determined. [2]

(*B*) Hence find the exact area of the region enclosed by the curve $y = \mathrm{f}(x)$, the positive x-axis and the line $x = 2$. [4]

12. Use the substitution $u = x + 1$ to find $\int (5x+2)\sqrt{x+1}\, \mathrm{d}x$. Give your answer in the form $kx(x + 1)^p + c$ where k, p and c are constants. [7]

13.

(a) Find $\int \left(\frac{x}{1+\sqrt{x}}\right) \mathrm{d}x$. You may use the substitution $u = 1 + \sqrt{x}$. [7]

(b) Hence show that $\int_0^1 \left(\frac{x}{1+\sqrt{x}}\right) \mathrm{d}x = A - \ln B$ where A and B are constants to be determined. [2]

END OF QUESTION paper

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	translation in the *x*-direction	M1	allow 'shift', 'move'	If just vectors given withhold **one** 'A' mark only
		i	of π/4 to the right	A1	oe (eg using vector)	'Translate $\begin{pmatrix}\pi/4\\1\end{pmatrix}$', is 4 marks; if this is followed by an additional incorrect transformation, SC M1M1A1A0
		i	translation in *y*–direction	M1	allow 'shift', 'move'	$\begin{pmatrix}\pi/4\\1\end{pmatrix}$ only is M2A1A0
		i	of 1 unit up.	A1	oe (eg using vector) **Examiner's Comments** We usually insist on the word 'translation' here, but in this case allowed 'move', 'shift', etc. A vector on its own does not in our view imply a translation. Occasionally, candidates clearly knew what the transformations were, but wrote the vectors incorrectly, for example the wrong way up. Nevertheless, this topic is usually well known and done well.	
		ii	$g(x)=\dfrac{2\sin x}{\sin x+\cos x}$			(Can deal with num and denom separately)
		ii	$g'(x)=\dfrac{(\sin x+\cos x)2\cos x-2\sin x(\cos x-\sin x)}{(\sin x+\cos x)^2}$	M1	Quotient (or product) rule consistent with their derivs	$\dfrac{vu'-uv'}{v^2}$; allow one slip, missing brackets
		ii	$=\dfrac{2\sin x\cos x+2\cos^2 x-2\sin x\cos x+2\sin^2 x}{(\sin x+\cos x)^2}$ $=\dfrac{2\cos^2 x+2\sin^2 x}{(\sin x+\cos x)^2}=\dfrac{2(\cos^2 x+\sin^2 x)}{(\sin x+\cos x)^2}$	A1	Correct expanded expression (could leave the '2' as a factor)	$\dfrac{uv'-vu'}{v^2}$
		ii	$=\dfrac{2}{(\sin x+\cos x)^2}$ *	A1	**NB AG**	must take out 2 as a factor or state $\sin^2 x+\cos^2 x=1$

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	When $x = \pi/4$, $g'(\pi/4) = 2/(1/\sqrt{2} + 1/\sqrt{2})^2$	M1	substituting π/4 into correct deriv	
		ii	= 1	A1		
		ii	$f'(x) = \sec^2 x$	M1	o.e., e.g. $1/\cos^2 x$	
		ii	$f'(0) = \sec^2(0) = 1$, [so gradient the same here]	A1	**Examiner's Comments** The quotient rule is generally well known, and errors here usually stemmed from faulty derivatives or poor algebra. Brackets are not optional in an expression like this, and their removal was not always successfully achieved. We also needed evidence of the use of $\cos^2 x + \sin^2 x = 1$, either by its direct quotation or by factoring out the '2' in the numerator. The evaluation of $g'(x)$ was usually correct. With $f'(x)$, some used a quotient rule on $\sin x/\cos x$ rather than quoting the derivative of $\tan x = \sec^2 x$; we also got some occasional 'translation' arguments here which misunderstood the nature of the verification.	
		iii	$= \int_1^{1/\sqrt{2}} -\frac{1}{u}\,du$ let $u = \cos x$, $du = -\sin x\,dx$ when $x = 0$, $u = 1$, when $x = \pi/4$, $u = 1/\sqrt{2}$			
		iii	$= \int_{1/\sqrt{2}}^{1} \frac{1}{u}\,du$ *	M1	substituting to get $\int -1/u$ (du)	ignore limits here, condone no du but not dx allow $\int 1/u.-du$
		iii	$= \int_{1/\sqrt{2}}^{1} \frac{1}{u}\,du$ *	A1	NB **AG**	but for A1 must deal correctly with the -ve sign by interchanging limits

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	$= [\ln u]_{1/\sqrt{2}}^{1}$	M1	[ln *u*]	
		iii	= ln 1 - ln (1/√2)			
		iii	= ln √2 = ln $2^{½}$ ln 2	A1	ln √2, ½ ln 2 or –ln(1/√2) **Examiner's Comments** This was a case where giving the transformed integral proved to be of doubtful value, as many candidates 'lost' the negative sign in their ∫-1/*u* d*u*, and placed the limits the wrong way round. It appears that the idea of swapping limits making the integral negative was not generally understood. The evaluation of the given integral with respect to *u* was more successfully done, though quite a few candidates approximated their final answer.	mark final answer
		iv	Area = area in part **(iii)** translated up 1 unit.	M1	soi from π/4 added	or
		iv	So = ½ ln 2 + 1 × π4 = ½ ln 2 + π/4.	A1cao	oe (as above) **Examiner's Comments** These marks were gained by candidates who managed to spot the rectangle of area added by the translation upwards of the graph of f(*x*).	
			Total	**17**		

Question			Answer/Indicative content	Marks	Part marks and guidance	
2			Let $u = 1 + x \Rightarrow$			
			$\int_0^3 x(1+x)^{-1/2}\,dx = \int_1^4 (u-1)u^{-1/2}\,du$	M1	$\int (u-1)u^{-1/2}(du)$*	condone no du, missing bracket, ignore limits
			$= \int_1^4 (u^{1/2} - u^{-1/2})\,du$	A1	$\int (u^{1/2} - u^{-1/2})(du)$	
			$= \left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^4$	A1	$\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]$ o.e.	e.g. $\left[\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right]$; ignore limits
			= (16/3 – 4) – (2/3 – 2)	M1dep	upper–lower dep 1st M1 and integration	with correct limits e.g. 1, 4 for u or 0, 3 for x
			$= 2\frac{2}{3}$	A1cao	or $2.\dot{6}$ but must be exact	or using $w = (1 + x)^{1/2} \Rightarrow \int \frac{(w^2-1)2w}{w}(dw)$ M1
			OR Let $u = x$, $v' = (1 + x)^{-1/2}$	M1		$= \int 2(w^2-1)(dw)$ A1 $= \left[\frac{2}{3}w^3 - 2w\right]$ A1
			$\Rightarrow u' = 1$, $v = 2(1 + x)^{1/2}$	A1		upper–lower with correct limits (w = 1,2) M1
			$\Rightarrow \int_0^3 x(1+x)^{-1/2}\,dx = \left[2x(1+x)^{1/2}\right]_0^3 - \int_0^3 2(1+x)^{1/2}\,dx$	A1	ignore limits, condone no dx	8/3 A1 cao
			$= \left[2x(1+x)^{1/2} - \frac{4}{3}(1+x)^{3/2}\right]_0^3$	A1	ignore limits	*If $\int_1^4 (u-1)u^{-1/2}\,du$ done by parts:
			= (2 × 3 × 2 – 4 × 8/3) – (0 – 4/3)			$2u^{1/2}(u-1) - \int 2u^{1/2}\,du$ A1 $[2u^{1/2}(u-1) - 4u^{3/2}/3]$ A1

Question			Answer/Indicative content	Marks	Part marks and guidance	
			$= 2\frac{2}{3}$	A1cao	or $2.\dot{6}$ but must be exact **Examiner's Comments** Most candidates used integration by substitution, though a significant minority used integration by parts. In general, the former were more successful, with the main difficulty being in expanding $(u - 1)u^{-1/2}$ as $u^{1/2} - u^{-1/2}$. Some proceeded from here using integration by parts, with mixed success. When parts were used, the most common error was in deriving $v = 2(1 + x)1/2$ from $v' = (1 + x)^{-1/2}$.	substituting correct limits M1 8/3 A1cao
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
3		i	$a = ½$	B1	allow $x = ½$ __Examiner's Comments__ Nearly all candidates gained this mark for the asymptote.	
		ii	$y^3 = \frac{x^3}{2x-1}$ $\Rightarrow \quad 3y^2\frac{dy}{dx} = \frac{(2x-1)3x^2 - x^3.2}{(2x-1)^2}$	B1 M1 A1	$3y^2 dy/dx$ Quotient (or product) rule consistent with their derivatives; (v du + udv)/v2 M0 correct RHS expression – condone missing bracket	
		ii	$= \frac{6x^3 - 3x^2 - 2x^3}{(2x-1)^2} = \frac{4x^3 - 3x^2}{(2x-1)^2}$	A1		
		ii	$\Rightarrow \quad \frac{dy}{dx} = \frac{4x^3 - 3x^2}{3y^2(2x-1)^2}$ *	A1	**NB AG** penalise omission of bracket in QR at this stage	
		ii	$dy/dx = 0$ when $4x^3 - 3x^2 = 0$	M1		
		ii	$\Rightarrow x^2(4x - 3) = 0, x = 0$ or $¾$	A1	if in addition $2x - 1 = 0$ giving $x = ½$, A0	
		ii	$y^3 = (¾)^3/½ = 27/32,$	M1	must use $x = ¾$; if (0, 0) given as an additional TP, then A0	
		ii	$y = 0.945$ (3sf)	A1	can infer M1 from answer in range 0.94 to 0.95 inclusive	
		ii	Additional suggestions $y = \frac{x}{(2x-1)^{1/3}}$ $\Rightarrow \frac{dy}{dx} = \frac{(2x-1)^{1/3}.1 - x.(1/3)(2x-1)^{-2/3}.2}{(2x-1)^{2/3}}$	 M1 A1	 quotient rule or product rule on y – allow one slip correct expression for the derivative	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$= \frac{6x-3-2x}{3(2x-1)^{4/3}} = \frac{4x-3}{3(2x-1)^{4/3}}$	M1 A1	factorising or multiplying top and bottom by $(2x-1)^{2/3}$	
		ii	$= \frac{(4x-3)x^2}{3y^2(2x-1)^{2/3}(2x-1)^{4/3}} = \frac{4x^3-3x^2}{3y^2(2x-1)^2}$	A1	establishing equivalence with given answer **NB AG**	
		ii	$y = \left(\frac{x^3}{(2x-1)}\right)^{1/3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{3}\left(\frac{x^3}{(2x-1)}\right)^{-2/3} \frac{(2x-1).3x^2 - x^3.2}{(2x-1)^2}$	B1 M1A1	$\frac{1}{3}\left(\frac{x^3}{(2x-1)}\right)^{-2/3} \times \ldots$ $\ldots \times \frac{(2x-1).3x^2 - x^3.2}{(2x-1)^2}$	
		ii	$= \frac{1}{3}\frac{4x^3-3x^2}{x^2(2x-1)^{4/3}} = \frac{4x-3}{3(2x-1)^{4/3}}$	A1		
		ii	$= \frac{(4x-3)x^2}{3y^2(2x-1)^{2/3}(2x-1)^{4/3}} = \frac{4x^3-3x^2}{3y^2(2x-1)^2}$	A1	establishing equivalence with given answer **NB AG**	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$y^3(2x-1)=x^3$ $3y^2\frac{dy}{dx}(2x-1)+y^3.2=3x^2$	B1	d/dx(y^3) = 3y^2(dy/dx)	
			$\frac{dy}{dx}=\frac{3x^2-2y^3}{3y^2(2x-1)}$	M1 A1	product rule on $y^3(2x-1)$ or $2xy^3$ correct equation	
			$=\frac{3x^2-2\frac{x^3}{(2x-1)}}{3y^2(2x-1)}$ $=\frac{3x^2(2x-1)-2x^3}{3y^2(2x-1)^2}=\frac{6x^3-3x^2-2x^3}{3y^2(2x-1)^2}=\frac{4x^3-3x^2}{3y^2(2x-1)^2}$	M1	subbing for $2y^3$	
				A1	**NB AG** ☐<u>**Examiner's Comments**</u> Candidates tended to score heavily on this part. The implicit differentiation of y^3 was usually correct (albeit introduced into solutions belatedly), and the quotient rule was done well, though occasionally omission of brackets was penalised. Those who cube rooted and differentiated often succeeded in arriving at the given derivative. Another approach was to multiplying across before differentiating implicitly, but with required candidates to substitute for y to deduce the required form for the derivative. Finding x = ¾ for the turning point from the given derivative was straightforward, but some failed to find the correct y-coordinate by omitting the necessary cube root.	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	$u = 2x - 1 \Rightarrow du = 2dx$ $\int \frac{x}{\sqrt[3]{2x-1}}dx = \int \frac{\frac{1}{2}(u+1)}{u^{1/3}}\frac{1}{2}du$	M1 M1	$\frac{\frac{1}{2}(u+1)}{u^{1/3}}$ if missing brackets, withhold A1 × ½ du condone missing du here, but withhold A1	
		iii	$= \frac{1}{4}\int \frac{u+1}{u^{1/3}}du = \frac{1}{4}\int (u^{2/3} + u^{-1/3})du$ *	A1	**NB AG**	
		iii	area $= \int_1^{4.5} \frac{x}{\sqrt[3]{2x-1}}dx$	M1	correct integral and limits – may be inferred from a change of limits and their attempt to integrate (their) ¼ $(u^{2/3} + u^{-1/3})$	
		iii	when x = 1, u = 1, when x = 4.5, u = 8	A1	u = 1, 8 (or substituting back to x's and using 1 and 4.5)	
		iii	$= \frac{1}{4}\int_1^8 (u^{2/3} + u^{-1/3})du$ $= \frac{1}{4}\left[\frac{3}{5}u^{5/3} + \frac{3}{2}u^{2/3}\right]_1^8$	B1	$\left[\frac{3}{5}u^{5/3} + \frac{3}{2}u^{2/3}\right]$ o.e. e.g. $[u^{5/3}/(5/3) + u^{2/3}/(2/3)]$	
		iii	$= \frac{1}{4}\left[\frac{96}{5} + 6 - \frac{3}{5} - \frac{3}{2}\right]$	A1	o.e. correct expression (may be inferred from a correct final answer)	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	$= 5\frac{31}{40} = 5.775$ or $\frac{231}{40}$	A1	cao, must be exact; mark final answer **Examiner's Comments** There were plenty of accessible marks here as well. The first three marks, for transforming the integral to the variable *u*, were usually negotiated successfully, although poor notation – omitting d*u*'s or brackets – was sometimes penalised in the A1 mark. The second half involved evaluating the given integral with the correct limits. Some calculated the correct limits, but made errors in the integral (or forgot to integrate altogether). However, a reasonable number of candidates managed to do this work without errors. A rather curious misconception was to cube the correct value of the integral, because the function was presented implicitly in terms of y^3.	
			Total	18		

Question			Answer/Indicative content	Marks	Part marks and guidance	
4			$\int_0^{\pi/2} \frac{\sin 2x}{3+\cos 2x} dx = \left[-\frac{1}{2}\ln(3+\cos 2x)\right]_0^{\pi/2}$	M1	$k \ln(3 + \cos 2x)$	
				A2	½ $\ln(3 + \cos 2x)$	
			or $u = 3 + \cos 2x$, $du = -2\sin 2x\, dx$	M1	o.e. e.g. $du/dx = -2\sin 2x$ or if $v = \cos 2x$, $dv = -2\sin 2x dx$ o.e. condone $2\sin 2x\, dx$	
			$\int_0^{\pi/2} \frac{\sin 2x}{3+\cos 2x} dx = \int_4^2 -\frac{1}{2u} du$ $= \left[-\frac{1}{2}\ln u\right]_4^2$ $= -$ ½ ln 2 + ½ ln 4 = ½ ln (4/2) = ½ ln 2 *	A1	$\int -\frac{1}{2u} du$, or if $v = \cos 2x$, $\int -\frac{1}{2(3+v)} dv$	
				A1	[– ½ ln$u$] or [– ½ ln(3 + v)] ignore incorrect limits	
				A1	from correct working o.e. e.g. – ½ ln(3 + cos(2.π/2)) + ½ ln(3 + cos(2.0)) o.e. required step for final A1, must have evaluated to 4 and 2 at this stage	
				A1	**NB AG** ☐<u>Examiner's Comments</u> The error d/dx (cos 2x) = 2sin 2x proved costly here, earning only a consolation M1; many also wrote the limits the wrong way round on the integral, and scored 3 out of 5, unless they 'lost' the negative sign, and scored M1 only. Many candidates seem unaware that swapping limits dealt with the negative sign. We also needed to see some evidence of why ln 4 – ln 2 = 2 to score the final A1.	

Question			Answer/Indicative content	Marks	Part marks and guidance
			Total	5	

Question			Answer/Indicative content	Marks	Part marks and guidance	
5		i	$f(-x)=\dfrac{-x}{\sqrt{2+(-x)^2}}$	M1	substituting –*x* for *x* in f (*x*)	$\dfrac{-x}{\sqrt{2+-x^2}},\dfrac{-x}{\sqrt{2+-(x^2)}},\dfrac{-x}{\sqrt{2+(-x^2)}}$ M1A0
		i	$=-\dfrac{x}{\sqrt{2+x^2}}=-f(x)$	A1	1st line must be shown, must have f (–*x*) = – f (*x*) oe somewhere	$\dfrac{-x}{\sqrt{2-x^2}}$ M0A0
		i	Rotational symmetry of order 2 about O	B1	must have 'rotate' and 'O' and 'order 2 or 180 or ½ turn' **Examiner's Comments** Most candidates stated that for an odd function f(-x) = -f(x) or equivalent. It is important when writing f(-x) that brackets are placed round the –x terms: if these were missing, the 'A' mark was lost. The structure of this 'show' was often a bit 'muddy': f(-x) = ... = ... = -f(x) is clear, but writing f(-x) = -f(x) and then writing expressions for each side of this equation below and showing they are equal is less so, as the direction of the argument, or implications, is not clear. The geometrical description of an odd function required three elements: 'rotational', 'order 2' and 'centre O' or equivalent; reflection in Ox followed by Oy was also allowed.	oe e.g. reflections in both *x*- and *y*-axes
		ii	$f'(x)=\dfrac{\sqrt{2+x^2}.1-x.\frac{1}{2}(2+x^2)^{-1/2}.2x}{(\sqrt{2+x^2})^2}$ $=\dfrac{2+x^2-x^2}{(2+x^2)^{3/2}}=\dfrac{2}{(2+x^2)^{3/2}}$ *	M1	quotient or product rule used	QR: condone *u*d*v* ± *v*d*u*, but *u*, *v* and denom must be correct
		ii		M1	½ $u^{-1/2}$ or – ½ $v^{-3/2}$ soi	
		ii		A1	correct expression	$x(-1/2)(2+x^2)^{-3/2}.2x+(2+x^2)^{-1/2}$. $=(2+x^{-3/2})(-x^2+2+x^2)$

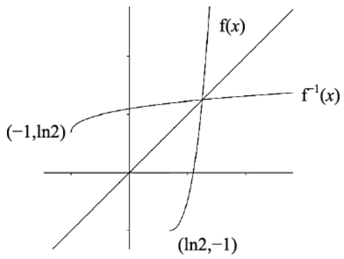
Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii		A1	**NBAG**	
		ii	When $x = 0$, $f'(x) = 2/2^{3/2} = 1/\sqrt{2}$	B1	oe e.g. $\sqrt{2}/2$, $2^{-1/2}$, $1/2^{1/2}$, but not $2/2^{3/2}$ **Examiner's Comments** The difficulty with this sort of product or quotient rule question lies in factorising and hence simplifying the expression, and this was the case here. Many wrote down correct expressions, but then failed to eshowf the printed answer. This difficulty often encouraged multiple attempts, sometimes using a quotient rule, followed by a product rule, etc. A surprising number of candidates muddled up their euf and evf and quotient and product rule, for example using v = (2+x2)– 1/2 in their quotient rule. Often the final answer failed to score because we insisted on this being simplified to 1/√2 or equivalent.	allow isw on these seen
		iii	$A = \int_0^1 \frac{x}{\sqrt{2+x^2}}[\mathrm{d}x]$	B1	correct integral and limits	limits may be inferred from subsequent working, condone no dx
		iii	let $u = 2 + x^2$, $\mathrm{d}u = 2x\,\mathrm{d}x$		or $v = \sqrt{(2 + x^2)}$, $\mathrm{d}v = x(2 + x^2)^{-1/2}\,\mathrm{d}x$	
		iii	$= \int_2^3 \frac{1}{2}\frac{1}{\sqrt{u}}\mathrm{d}u$	M1	$\int \frac{1}{2}\frac{1}{\sqrt{u}}[\mathrm{d}u]$ or $= \int 1[\mathrm{d}v]$ or $k(2+x^2)^{1/2}$	condone no du or dv, but not $\int \frac{1}{2}\frac{1}{\sqrt{u}}\mathrm{d}x$
		iii	$= \left[u^{1/2}\right]_2^3$	A1	$[u^{1/2}]$ o.e. (but not $1/u^{-1/2}$) or $[v]$ or $k = 1$	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	$= \sqrt{3} - \sqrt{2}$	A1cao	must be exact **Examiner's Comments** A substantial minority of candidates thought this integral should be done by parts, and therefore scored nothing after the first B1. Those who tried substituting often got muddled before arriving at $\int 1/2\sqrt{u}\, du$, and some then integrated this incorrectly, e.g as $\ln\sqrt{u}$	isw approximations
		iv	$y^2 = \dfrac{x^2}{2+x^2}$	M1	squaring (correctly)	must show $[\sqrt{(2 + x^2)}]^2 = 2 + x^2$ (o.e.)
		iv	$\Rightarrow 1/y^2 = (2 + x^2)/x^2 = 2/x^2 + 1$ *	A1	or equivalent algebra **NB AG** **Examiner's Comments** This simple piece of algebra was often over-complicated by round-the-houses methods. An all- too-commonly seen mistake was $x^2/(2+x^2) = x^2/2 + 1$.	If argued backwards from given result without error, SCB1
		iv	$-2y^{-3}dy/dx = -4x^{-3}$	B1B1	LHS, RHS	condone $dy/dx - 2y^{-3}$ unless pursued
		iv	$\Rightarrow dy/dx = -4x^{-3}/-2y^{-3} = 2y^3/x^3$*	B1	**NB AG**	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iv	Not possible to substitute $x = 0$ and $y = 0$ into this expression	B1	soi (e.g. mention of 0/0) **Examiner's Comments** The implicit differentiation was usually correct, as was the algebra to arrive at the printed result. The exact logic behind why x = 0 and y = 0 could not be substituted into the result expression was often faulty (for example many stated the result would be zero or infinite); we condoned this provided they stated the idea that division by zero is undefined or not possible.	Condone 'can't substitute $x = 0$' o.e. (i.e. need not mention $y = 0$). Condone also 'division by 0 is infinite'
		v	$-2y^{-3}\mathrm{d}y/\mathrm{d}x = -4x^{-3}$ $\Rightarrow \mathrm{d}y/\mathrm{d}x = -4x^{-3}/-2y^{-3} = 2y^3/x^3$* Not possible to substitute $x = 0$ and $y = 0$ into this expression		LHS, RHS **NB AG** soi (e.g. mention of 0/0)	condone $\mathrm{d}y/\mathrm{d}x - 2y^{-3}$ unless pursued Condone 'can't substitute $x = 0$' o.e. (i.e. need not mention $y = 0$). Condone also 'division by 0 is infinite'
			Total	18		

Question			Answer/Indicative content	Marks	Part marks and guidance	
6		i	At P, $(e^x – 2)^2 – 1 = 0$			
		i	$\Rightarrow e^x – 2 = [\pm]1$,	M1	square rooting – condone no ±	
		i	$e^x = [1 \text{ or}]\ 3$			
		i	*or* $(e^x)^2 – 4e^x + 3 = 0$	M1	expanding to correct quadratic and solve by factorising or using quadratic formula	condone e^*x*^2
		i	$\Rightarrow (e^x – 1)(e^x – 3) = 0$, $e^x = 1$ or 3			
		i	$\Rightarrow x = [0 \text{ or}]\ \ln 3$	A1	*x*-coordinate of P is ln 3; must be exact **Examiner's Comments** Most candidates succeeded in finding x = ln 3, either by square rooting or solving thequadratic in e^x. The second method was somewhat compromised by setting $x = e^x$ (ratherthan a different variable) to get a quadratic in x, though we condoned this for both marks.	condone P = ln 3, but not *y* = ln 3
		ii	$f'(x) = 2(e^x – 2)e^x$	M1	chain rule	e.g. 2 *u* × their deriv of e^x
		ii		A1	correct derivative	$2(e^x – 2)x$ is M0
		ii	$= 0$ when $e^x = 2$, $x = \ln 2$ *	A1	not from wrong working **NB AG**	or verified by substitution
		ii	*or* $f(x) = e^{2x} – 4e^x + 3$	M1	expanding to 3 term quadratic with $(e^x)^2$ or e^{2x}	condone e^*x*^2
		ii	$\Rightarrow f'(x) = 2e^{2x} – 4e^x$	A1	correct derivative, not from wrong working	
		ii	$= 0$ when $2e^{2x} = 4e^x$, $e^x = 2$, $x = \ln 2$ *	A1	or $2e^x(e^x – 2) = 0 \Rightarrow e^x = 2$, $x = \ln 2$	or verified by substitution
		ii			not from wrong working **NB AG**	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$y = f(\ln(2)) = -1$	B1	**Examiner's Comments** This provided a simple four marks for most candidates, using a chain rule to find the derivative, setting this to zero and solving to get x = ln 2. A neat alternative method was to recognise that the $(e^x - 2)^2$ term must be non-negative and minimum when $e^x - 2 = 0$, or x = ln 2.	
		iii	$\int_0^{\ln 3}[(e^x-2)^2-1]dx$ $=\int_0^{\ln 3}[(e^x)^2-4e^x+4-1]dx$	M1	expanding brackets must have 3 terms: $(e^x)^2 - 4$ is M0, condone e^x^2	or if $u = e^x$, $\int_1^3[u^2 - 4u + 4 - 1]/u\,du$
		iii	$=\int_0^{\ln 3}[e^{2x}-4e^x+3]dx$	A1	$\int e^{2x} - 4e^x + 3$ [dx] (condone no dx)	$= \int u - 4 + 3/u\,du$
		iii	$=\left[\frac{1}{2}e^{2x}-4e^x+3x\right]_0^{\ln 3}$	B1	$\int e^{2x} = ½ e^{2x}$	= [½ $u^2 - 4u + 3\ln u$
		iii		A1ft	[½ $e^{2x} - 4e^x + 3x$]	
		iii	= (4.5 – 12 + 3ln3) – (0.5 – 4)			
		iii	= 3ln3 – 4 [so area = 4 – 3ln3]	A1	condone 3ln3 – 4 as final ans; mark final ans **Examiner's Comments** This proved to be a rather costly part for candidates unless they recognised the requirement to multiply out $(e^x - 2)^2 - 1$ to get $e^{2x} - 4e^x + 3$ and then integrate term-by-term. Other attempts using substitution or parts usually got nowhere. Although originally we required candidates to give the area as 4 – 3ln3, very few actually did this, so it was decided to condone a (negative) area of 3ln3 – 4.	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iv	$y = (e^x - 2)^2 - 1$ $x \leftrightarrow y$			
		iv	$x = (e^y - 2)^2 - 1$			
		iv	$\Rightarrow x + 1 = (e^y - 2)^2$	M1	attempt to solve for y (might be indicated by expanding and then taking lns)	or x if x and y not interchanged yet or adding (or subtracting) 1
		iv	$\Rightarrow \pm\sqrt{(x + 1)} = ey - 2$ (+ for $y \geq \ln 2$)	A1	condone no ±	
		iv	$\Rightarrow 2 + \sqrt{(x + 1)} = e^y$			
		iv	$\Rightarrow y = \ln(2 + \sqrt{(x + 1)}) = f^{-1}(x)$	A1	must have interchanged x and y in final ans	
		iv	Domain is $x \geq -1$	B1	must be ≥ and x (not y)	if not specified, assume first ans is domain and second range
		iv	Range is $y \geq \ln 2$	B1	or $f^{-1}(x) \geq \ln 2$, must be ≥ (not x or $f(x)$) if $x > -1$ and $y > \ln 2$ SCB1	
		iv		M1	recognisable attempt to reflect curve, or any part of curve, in $y = x$	$y = x$ shown indicative but not essential

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iv		A1	good shape, cross on $y = x$ (if shown), correct domain and range indicated. [see extra sheet for examples] **Examiner's Comments** Rather more than half of the candidates managed the inverse function well, though a few made errors at the last stage of taking the square root, and concluded with y = ln($\sqrt{(x+1)}$)+ 2, or y = ln($\sqrt{(x+1)}$+ ln 2. Some were perhaps encouraged by the previous part to multiply out $(e^x - 2)^2$ again, though they could still obtain a method mark for a step towards finding y in terms of x. It was not uncommon to see candidates taking logs of individual terms.	e.g. – 1 and ln 2 marked on axes
			Total	18		

Question			Answer/Indicative content	Marks	Part marks and guidance	
7			let $u = 2x - 1$, $du = 2\,dx$			
			$\int \sqrt[3]{2x-1}\,dx = \int \frac{1}{2}u^{\frac{1}{3}}\,du$	M1	substituting $u = 2x - 1$ in integral	i.e. $u^{1/3}$ or $\sqrt[3]{u}$ seen in integral
				M1	× ½ o.e.	condone no du, or dx instead of du
			$= \frac{3}{8}u^{\frac{4}{3}} + c$	M1	integral of $u^{1/3} = u^{4/3}/(4/3)$ (oe) soi	not $x^{1/3}$
			$= \frac{3}{8}(2x-1)^{\frac{4}{3}} + c$	A1cao	o.e., but must have + c and single fraction mark final answer	so $\frac{3}{4}(2x-1)^{\frac{4}{3}} + c$ is M1M0M1A0
			or			
			$\int \sqrt[3]{2x-1}\,dx = ½ \times (2x-1)^{4/3} \div 4/3$	M1	$(2x - 1)^{4/3}$ seen	e.g. correct power of $(2x - 1)$
				M1	÷ 4/3 (oe) soi	e.g. ¾ $(2x - 1)^{4/3}$ seen
				M1	× ½	
			$= \frac{3}{8}(2x-1)^{\frac{4}{3}} + c$	A1cao	o.e., but must have + c and single fraction mark final ans **Examiner's Comments** This question was also answered well, either using substitution or by inspection. However, a surprising number of candidates who substituted left their final answer in terms of u, and a few lost the final mark through omitting the arbitrary constant.	so $\frac{3}{8}(2x-1)^{\frac{4}{3}}$ is M1M1M1A0
			Total	4		

Question			Answer/Indicative content	Marks	Part marks and guidance	
8		i	$\frac{dy}{dx} = \frac{(x+4)^{1/2}.1 - x.\frac{1}{2}(x+4)^{-1/2}}{[(x+4)^{1/2}]^2}$	M1	quotient rule: v × their u' – u × their v', and correct denominator	or product rule
		i		B1	½ $u^{-1/2}$ soi	or –½ $u^{-3/2}$ (PR)
		i		A1	correct expression	PR: $x(-½)(x+4)^{-3/2} + (x+4)^{-1/2}$
		i	$= \frac{x+4-\frac{1}{2}x}{(x+4)^{3/2}} = \frac{\frac{1}{2}x+4}{(x+4)^{3/2}} = \frac{x+8}{2(x+4)^{3/2}}$ *	M1	factoring out $(x+4)^{-1/2}$ o.e.	$= (x+4)^{-3/2}(-½x + x + 4)$
		i		A1	NB AG	**Examiner's Comments** Most candidates scored well on this question, which covered calculus topics such as the product or quotient rule for differentiation and integration by substitution, which are generally well understood by learners. The first three marks here were usually earned, though a minority of weaker candidates mixed up the product and quotient rules, for example using $v = (x+4)^{-1/2}$ in their quotient rule. The factorisation required to achieve the given result was less successfully done, but just over half the candidates still managed full marks here. There were a lot of repeated attempts at this, for example using the product rule when they got stuck with manipulating their quotient rule expression.
		ii	[asymptote is] $x = -4$	B1	soi	but from correct working
		ii	gradient of tangent at O= $8/(2 \times 4^{3/2}) = ½$	B1	gradient = ½	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	eqn of tangent is $y = ½x$	B1	o.e. e.g. using gradient	
		ii	When $x = –4$, $y = –2$, so (–4, –2)	B1		**Examiner's Comments** Most candidates scored well on this question, which covered calculus topics such as the product or quotient rule for differentiation and integration by substitution, which are generally well understood by learners. This proved to be a straightforward 4 marks earned by over 70% of scripts. The asymptote and the gradient and equation of the tangent at the origin were usually correctly found, followed by the coordinates of Q.
		iii	let $u = x + 4$, $du = dx$	B1	or $dx/du = 1$	or $v^2 = x + 4$, $2v dv/dx = 1$ or $2v dv = dx$ oe e.g. $dv/dx = ½(x + 4)^{–1/2}$
		iii	$\int_0^5 \frac{x}{(x+4)^{1/2}}\,dx = \int_4^9 \frac{u-4}{u^{1/2}}\,du$	B1	$\int \frac{u-4}{u^{1/2}}[du]$	
		iii	$= \int_4^9 (u^{1/2} - 4u^{-1/2})\,du$	B1	$u^{1/2} – 4u^{–1/2}$ or $u^{1/2} – 4/u^{1/2}$, or $\sqrt{u} – 4/\sqrt{u}$	$\int (2v^2 – 8)[dv]$
		iii	$= \left[\frac{2}{3}u^{3/2} - 8u^{1/2}\right]_4^9$	B1	$\left[\frac{2}{3}u^{3/2} - 8u^{1/2}\right]$ o.e.	
		iii	= (18 – 24) – (16/3 – 16)	M1	substituting correct limits (upper – lower)	0, 5 for x; 4,9 for u; 2,3 for v
		iii	= 14/3	A1cao		
		iii	**or** (following first 2 marks)			by parts with no substitution:
		iii	let $v = u – 4$, $w' = u^{–1/2}$, $v' = 1$, $w = 2u^{1/2}$	M1		$u = x$, $u' = 1$, $v' = (x + 4)^{–1/2}$, $v = 2(x + 4)^{1/2}$ M1 $= [2x(x + 4)^{1/2}] – \int 2(x + 4)^{1/2}$ A1

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	$\int_4^9 (u-4)u^{-1/2}\,\mathrm{d}u = \left[2u^{1/2}(u-4)\right]_4^9 - \int_4^9 2u^{1/2}\,\mathrm{d}u$	A1		
		iii	$= \left[2u^{1/2}(u-4) - \frac{4}{3}u^{3/2}\right]_4^9$	A1		= 14/3 A1 (so max of 4/6)
		iii	= 14/3	A1cao		
		iii	*y*- coordinate of Q is 2½	B1	(soi)	or $\int_0^5 \frac{1}{2}x\,\mathrm{d}x$ M1
		iii	Area of triangle = ½ × 5 × 5/2 = 25/4	B1		$= \left[\frac{1}{4}x^2\right]_0^5 = 25/4$ A1

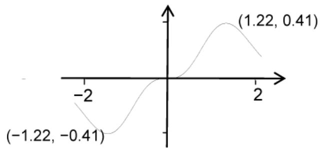
Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	Enclosed area = 25/4 – 14/3 = $1\frac{7}{12}$	B1	or 19/12, or $1.58\dot{3}$	isw from correct exact answer **Examiner's Comments** Most candidates scored well on this question, which covered calculus topics such as the product or quotient rule for differentiation and integration by substitution, which are generally well understood by learners. This 9-mark question required careful extended work from candidates, but there was a pleasing response, with just under half the scripts earning full marks. The first six of these were for finding the area under the function using substitution. Here, as usual, notation sometimes left something to be desired, with missing d*u*'s or d*x*'s, integral signs, inconsistent limits, etc. Most of this we condoned, but we did require d*u*/d*x* = 1 or its equivalent to be stated. The final three marks depended upon the correct coordinates for the point Q being found in part (ii). Occasionally the triangle area was found using $\int \frac{1}{2} x \, dx$.
			Total	18		

Question			Answer/Indicative content	Marks	Part marks and guidance		
9			let $u=1+\sqrt{x}$ $\quad \mathrm{d}u=\frac{1}{2}x^{-\frac{1}{2}}\,\mathrm{d}x$ $\Rightarrow$ dx = 2 (u – 1)du $\Rightarrow \int_0^1 \frac{1}{1+\sqrt{x}}\,\mathrm{d}x=\int_1^2 \frac{2(u-1)}{u}\,\mathrm{d}u$ $=\int_1^2\left(2-\frac{2}{u}\right)\mathrm{d}u$ $=\left[2u-2\ln u\right]_1^2$ = 4 – 2ln 2 – 2 = 2 – 2ln 2 or 2 – ln 4	M1(AO3.1a) A1(AO1.1) A1(AO1.1) M1(AO3.1a) A1(AO1.1) A1cao(AO1.1) [6]	substituting $u=1+\sqrt{x}$ or $w=\sqrt{x}$ dx = 2(u – 1) du or dx = 2w dw $\frac{2(u-1)}{u}(\mathrm{d}u)$ or $\frac{2w}{(w+1)}(\mathrm{d}w)$ splitting fraction or dividing to get $2-\frac{2}{(w+1)}$ (or substituting $u = w + 1 \Rightarrow \frac{2(u-1)}{u}$ and then splitting fraction) or $[2w-2\ln(w+1)]_0^1$ if still in terms of w	Evidence of method must be seen Evidence of method must be seen	
			Total	6			

Question			Answer/Indicative content	Marks	Part marks and guidance		
10			$\frac{du}{dx} = 3$	B1(AO 1.1)		Other correct methods eg integration by parts, are acceptable	
			substitution of $3x = u - 1$	M1(AO 1.1)			
			$\int 2(u-1)u^7 \, du$	A1(AO 1.1)			
			$\frac{2u^9}{9} - \frac{2u^8}{8} (+c)$	A1(AO 1.1) A1(AO 1.1)	for either term correctly integrated both correct		
			$\frac{2}{9}(3x+1)^9 - \frac{1}{4}(3x+1)^8 + c$	A1(AO 1.1)			
				[6]			
			Total	6			

Question			Answer/Indicative content	Marks	Part marks and guidance		
11		i	$f(-x) = (-x)^3 e^{-(-x)^2}$ $= -x^3 e^{-x^2} = -f(x)$ Rotational symmetry of order two about the origin.	M1 A1 B1 [3]	substituting $-x$ for x in $f(x)$ must have $f(-x) = (-x)^3 e^{-(-x)^2}$ for A1 or point or half-turn (180°) symmetry about O	at least once allow description of symmetry, e.g. 'fits its outline if rotated etc…'	
					Examiner's Comments Most candidates scored 2 or 3 here. We required to see $f(-x) = (-x)^3 \exp(-x)^2$ in the proof that f(x) was an odd function, with the brackets correctly placed. For the 'B' mark describing the property of the graph, we needed to see reference to 'symmetry', 'half-turn, 180° or order 2', and 'about the origin'.		
		ii	$f'(x) = 3x^2 e^{-x^2} + x^3 (-2x) e^{-x^2}$ $f'(x) = 0$ when $3x^2 e^{-x^2} - 2x^4 e^{-x^2} = 0$ $\Rightarrow 3x^2 = 2x^4$ $\Rightarrow x = 0, \sqrt{1.5}, -\sqrt{1.5}$ $y = 0, 0.41. -0.41$ So (0, 0), (1.22, 0.41), (–1.22, –0.41)	M1 A1* M1 M1 A1dep A2dep	product rule correct expression their deriv = 0 taking out or dividing by e^{-x^2} dep A1*	consistent with their derivatives - condone deriv of e^{-x^2} is e^{-x^2} for M1 must be 2 terms must be 2 terms	

Question			Answer/Indicative content	Marks	Part marks and guidance		
				[7]	or $x = \pm\sqrt{1.5}$ o.e. dep A1*	Allow SC A1 if both x-coords correct or one point correct (dep A1*)	
					Examiner's Comments The main problem with the product rule here was to get the correct derivative of $\exp(-x^2)$. A common mistake was to think this is $\exp(-x^2)$. Having found the correct derivative and equated it to zero, the next issue was dividing through by, or factorising, $\exp(-x^2)$. After this, not many candidates got all three turning points, either omitting the origin or (–1.22, –0.41) or both. Also, evaluating the y -coordinates was sometimes done incorrectly. Where these issues were overcome, half of the candidates scored 6 or over; of these, half scored full marks.		

Question			Answer/Indicative content	Marks	Part marks and guidance		
		iii		M1 A1dep [2]	correct shape for $-2 \leq x \leq 2$ with 2 TPs, through O, reasonable half turn symmetry coords of TPs and stationary inflexion at origin shown dep (0,0) given as a stationary point in part (ii) **Examiner's Comments** Very few candidates scored both marks here. Many omitted the inflection at the origin, and the graphs were often lacking the point symmetry stated in part (i).	need not show stationary inflexion at O. ignore shape outside $-2 \leq x \leq 2$ condone plotting beyond [–2, 2] provided shape is correct	
		iv	(*A*) let $t = x^2$, $\mathrm{d}t/\mathrm{d}x = 2x$ [$\Rightarrow x\mathrm{d}x = ½\,\mathrm{d}t$] o.e. $\int x^3 e^{-x^2}[\mathrm{d}x] = \int x^2 e^{-x^2} x[\mathrm{d}x] = \int \frac{1}{2} t e^{-t}[\mathrm{d}t]$	M1 A1 [2]	$k = ½$ **Examiner's Comments** Half the candidates scored these two marks. Using a substitution in this context was perhaps unexpected.		

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iv	(B) $\int_0^2 x^3 e^{-x^2}\,\mathrm{d}x = k\int_0^4 te^{-t}\,\mathrm{d}t$			
			let $u = t$, $v' = e^{-t}$, $u' = 1$, $v = -e^{-t}$	M1	correct parts on $\int te^{-t}[dt]$ or $\int kte^{-t}[dt]$	ft their k, condone $v = e^{-t}$
			$=[k]\left\{\left[t(-e^{-t})\right]_0^4 - \int_0^4(-e^{-t})\,\mathrm{d}t\right\}$	A1	ignore limits, ft their k	
			$=[k]\left\{\left[-e^{-t} - te^{-t}\right]_0^4\right\}$	A1	limits must be correct here, ft their k	
			$=-\frac{1}{2}e^{-4} - 2e^{-4} + \frac{1}{2} = \frac{1}{2} - \frac{5}{2e^4}$	A1cao	oe but must evaluate $e^0 = 1$ and combine e^{-4} terms	
				[4]	**Examiner's Comments** They could get three out of the four marks with a missing, or incorrect, value for k, but not many succeeded with this.	
			Total	18		

Question			Answer/Indicative content	Marks	Part marks and guidance	
12			$\frac{\mathrm{d}u}{\mathrm{d}x}=1$	B1 (AO 1.1a)	Or du = dx	
			$\int(5u-3)u^{\frac{1}{2}}\,\mathrm{d}u$	M1 (AO 1.1)	Complete substitution for x and dx	
			$\int\left(5u^{\frac{3}{2}}-3u^{\frac{1}{2}}\right)\mathrm{d}u$	M1 (AO 1.1)		
			$2u^{\frac{5}{2}}-2u^{\frac{3}{2}}\ [+c]$	M1 (AO 1.1)		
			$2(x+1)^{\frac{5}{2}}-2(x+1)^{\frac{3}{2}}+c$	M1 (AO 1.1)		
			$\{2(x+1)-2\}(x+1)^{\frac{3}{2}}+c$	M1 (AO 1.1)	Taking out factor $(x+1)^{\frac{3}{2}}$	
			$2x(x+1)^{\frac{3}{2}}+c$	A1 (AO 2.1) [7]	Correct answer in correct form	
			Total	7		

Question			Answer/Indicative content	Marks	Part marks and guidance
13		a	$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}$	B1 (AO1.1)	
			$x = (u - 1)^2$	M1 (AO2.1)	allow sign error
			$\int \frac{(u-1)^2}{u} \times 2(u-1)\ \mathrm{d}u$	M1 (AO3.1a)	FT their x and their derivative
			$2\int \left(u^2 - 3u + 3 - \frac{1}{u}\right) \mathrm{d}u$ oe	A1 (AO1.1)	must be in a form ready to integrate
			$2\left[\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right]$	A1 (AO1.1) A1 (AO1.1)	three terms correct all four terms correct
			$\frac{2(1+\sqrt{x})^3}{3} - 3(1+\sqrt{x})^2 + 6(1+\sqrt{x}) - 2\ln(1+\sqrt{x})$ $[+c]$	A1 (AO3.2a) [7]	allow full marks if + c omitted
		b	Evaluation of F[1] – F[0]	M1 (AO2.1)	
			$\frac{5}{3}$ – in 4	A1 (AO1.1) [2]	
			Total	9	