Questions

Q1.

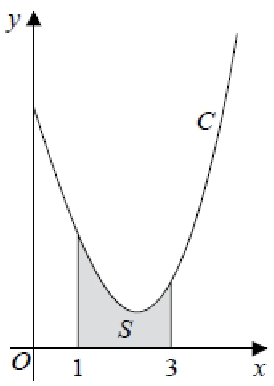

Figure 4

Figure 4 shows a sketch of part of the curve *C* with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region *S*, shown shaded in Figure 4, is bounded by the curve *C*, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of *S*, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of *S*.

(1)

(c) Show that the exact area of *S* can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

(*In part c*, solutions based entirely on graphical or numerical methods are not acceptable.)

(Total for question = 10 marks)

Q2.

The speed of a small jet aircraft was measured every 5 seconds, starting from the time it turned onto a runway, until the time when it left the ground.

The results are given in the table below with the time in seconds and the speed in ms^{-1}.

Time (s)	0	5	10	15	20	25
Speed ($m s^{-1}$)	2	5	10	18	28	42

Using all of this information,

(a) estimate the length of runway used by the jet to take off.

(3)

Given that the jet accelerated smoothly in these 25 seconds,

(b) explain whether your answer to part (a) is an underestimate or an overestimate of the length of runway used by the jet to take off.

(1)

(Total for question = 4 marks)

Q3.

The table below shows corresponding values of *x* and *y* for $y = \sqrt{\frac{x}{1+x}}$

The values of *y* are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\,\mathrm{d}x$$

giving your answer to 3 significant figures.

(3)

(b) Using your answer to part (a), deduce an estimate for $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\,\mathrm{d}x$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\,\mathrm{d}x = 4.535 \text{ to 4 significant figures}$$

(c) comment on the accuracy of your answer to part (b).

(1)

(Total for question = 5 marks)

Q4.

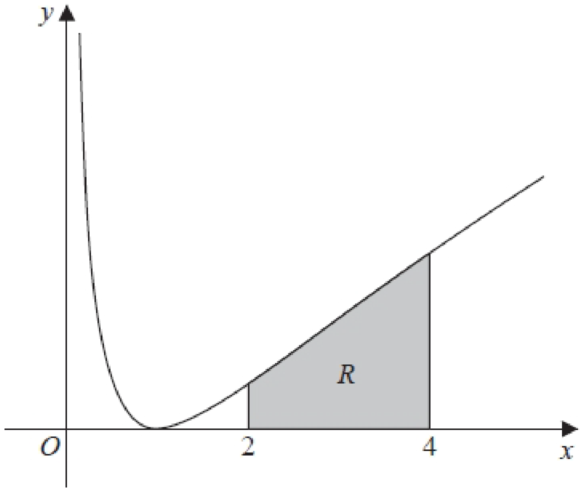

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \qquad x > 0$$

The finite region *R*, shown shaded in Figure 2, is bounded by the curve, the line with equation *x* = 2, the *x*-axis and the line with equation *x* = 4

The table below shows corresponding values of *x* and *y*, with the values of *y* given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

(a) Use the trapezium rule, with all the values of *y* in the table, to obtain an estimate for the area of *R*, giving your answer to 3 significant figures.

(3)

(b) Use algebraic integration to find the exact area of *R*, giving your answer in the form

$$y = a(\ln 2)^2 + b\ln 2 + c$$

where *a*, *b* and *c* are integers to be found.

(5)

(Total for question = 8 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Uses or implies $h = 0.5$	B1	1.1b
	For correct form of the trapezium rule =	M1	1.1b
	$\frac{0.5}{2}\{3+2.2958+2(2.3041+1.9242+1.9089)\}=4.393$	A1	1.1b
		(3)	
(b)	Any valid statement reason, for example • Increase the number of strips • Decrease the width of the strips • Use more trapezia	B1	2.4
		(1)	
(c)	For integration by parts on $\int x^2 \ln x \, dx$	M1	2.1
	$=\frac{x^3}{3}\ln x-\int \frac{x^2}{3}\, dx$	A1	1.1b
	$\int -2x+5\, dx=-x^2+5x \quad (+c)$	B1	1.1b
	All integration attempted and limits used Area of $S=\int_1^3 \frac{x^2 \ln x}{3}-2x+5\, dx=\left[\frac{x^3}{9}\ln x-\frac{x^3}{27}-x^2+5x\right]_{x=1}^{x=3}$	M1	2.1
	Uses correct ln laws, simplifies and writes in required form	M1	2.1
	Area of $S=\frac{28}{27}+\ln 27 \quad (a=28, b=27, c=27)$	A1	1.1b
		(6)	
			(10 marks)

Notes:
(a)
B1: States or uses the strip width $h = 0.5$. This can be implied by the sight of $\frac{0.5}{2}\{...\}$ in the trapezium rule
M1: For the correct form of the bracket in the trapezium rule. Must be y values rather than x values $\{\text{first } y \text{ value} + \text{last } y \text{ value} + 2\times(\text{sum of other } y \text{ values})\}$
A1: 4.393
(b)
B1: See scheme
(c)
M1: Uses integration by parts the right way around. Look for $\int x^2 \ln x \, dx = Ax^3 \ln x - \int Bx^2 \, dx$
A1: $\frac{x^3}{3}\ln x - \int \frac{x^2}{3} \, dx$
B1: Integrates the $-2x+5$ term correctly $= -x^2 + 5x$
M1: All integration completed and limits used
M1: Simplifies using ln law(s) to a form $\frac{a}{b} + \ln c$
A1: Correct answer only $\frac{28}{27} + \ln 27$

Q2.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$(\frac{1}{2} \times 5)(42 + 2 + 2 \times (5 + 10 + 18 + 28))$	M1	This mark is given for a method to use the trapezium rule as an approximation to the area under the curve
		M1	This mark is given for a correct terms used for the trapezium rule
	415 m	A1	This mark is given for a correct estimate of the length of the runway
(b)	An overestimate since the area of the five trapezia is greater than the area under the curve	B1	This mark is given for a valid explanation
			(Total 4 marks)

Q3.

Question	Scheme	Marks	AOs
(a)	$h = 0.5$	B1	1.1a
	$A \approx \frac{0.5}{2}\{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$	M1	1.1b
	= awrt 1.50	A1	1.1b
	For reference: **The integration on a calculator gives 1.511549071** **The full accuracy for y values gives 1.504726147** **The accuracy from the table gives 1.50475**		
		(3)	

(b)	$3 \times$ their (a)
	If (a) is correct, allow awrt 4.50 or awrt 4.51 even with no working.
	Only allow 4.5 if (a) is correct and working is shown e.g. 3×1.5
	
	If (a) is incorrect allow $3 \times$ their (a) given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a))
		B1ft	2.2a

	For reference the integration on a calculator gives 4.534647213		
		(1)	

Question	Scheme	Marks	AOs
(c)	**<u>This mark depends on the B1 having been awarded in part (b) with awrt 4.5</u>** Look for a sensible comment. Some examples: • The answer is accurate to 2 sf or one decimal place • Answer to (b) is accurate as $4.535 \approx 4.50$ • Very accurate as 4.535 to 2 sf is 4.5 • $4.51425 < 4.535$ so my answer is underestimate but not too far off • It is an underestimate but quite close • It is a very good estimate • High accuracy • (Quite) accurate • It is less than 1% out • $4.535 - 4.5 = 0.035$ so not far out But **not** just "it is an underestimate" or Calculates percentage error correctly using awrt 4.50 or awrt 4.51 or 4.5 (No comment is necessary in these cases although one may be given) Examples: $\left\lvert\frac{4.535-4.50}{4.535}\right\rvert \times 100 = 0.77\%$ or $\left\lvert\frac{4.535-4.51}{4.535}\right\rvert \times 100 = 0.55\%$ or $\left\lvert\frac{4.535-4.51425}{4.535}\right\rvert \times 100 = 0.46\%$ or $\left\lvert\frac{4.50}{4.535}\right\rvert \times 100 = 99\%$ In these cases don't be too concerned about accuracy e.g. allow 1sf. **This mark should be withheld if there are any contradictory statements**	B1	3.2b
		(1)	
			(5 marks)

<u>Notes:</u>

(a)

B1: States or uses $h = 0.5$. May be implied by $\frac{1}{4} \times \{\ldots$ below.

M1: Correct attempt at the trapezium rule.

Look for $\frac{1}{2}h \times \{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$ condoning slips on the terms but must use all y values with no repeats.

There must be a clear attempt at $\frac{1}{2}h \times$(first y + last y + $2\times$"sum of the rest")

Give M0 for $\frac{1}{2} \times \frac{1}{2} 0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)$ unless the missing brackets are implied.

NB this incorrect method gives 5.85…

May be awarded for separate trapezia e.g.

$$\frac{1}{4}(0.5774 + 0.7071) + \frac{1}{4}(0.7071 + 0.7746) + \frac{1}{4}(0.7746 + 0.8165) + \frac{1}{4}(0.8165 + 0.8452)$$

May be awarded for using the function e.g. $\frac{1}{2}h \times \left\{\sqrt{\frac{0.5}{1+0.5}} + \sqrt{\frac{2.5}{1+2.5}} + 2\left(\sqrt{\frac{1}{1+1}} + \sqrt{\frac{1.5}{1+1.5}} + \sqrt{\frac{2}{1+2}}\right)\right\}$

A1: Awrt 1.50 (Apply isw if necessary)

Correct answers with no working – send to review

(b)

B1ft: See main scheme. Must be considering $3 \times$ (a) and not e.g. attempting trapezium rule again.

(c)

B1: See scheme

Q4.

Question	Scheme	Marks	AOs
(a)	$h = 0.5$	B1	1.1b
	$A \approx \frac{1}{2} \times \frac{1}{2}\{0.4805 + 1.9218 + 2(0.8396 + 1.2069 + 1.5694)\}$	M1	1.1b
	$= 2.41$	A1	1.1b
		(3)	
(b)	$\int (\ln x)^2 \, dx = x(\ln x)^2 - \int x \times \frac{2\ln x}{x} \, dx$	M1 A1	3.1a 1.1b
	$= x(\ln x)^2 - 2\int \ln x \, dx = x(\ln x)^2 - 2\left(x\ln x - \int dx\right)$ $= x(\ln x)^2 - 2\int \ln x \, dx = x(\ln x)^2 - 2x\ln x + 2x$	dM1	2.1
	$\int_2^4 (\ln x)^2 \, dx = \left[x(\ln x)^2 - 2x\ln x + 2x\right]_2^4$ $= 4(\ln 4)^2 - 2\times 4\ln 4 + 2\times 4 - \left(2(\ln 2)^2 - 2\times 2\ln 2 + 2\times 2\right)$ $= 4(2\ln 2)^2 - 16\ln 2 + 8 - 2(\ln 2)^2 + 4\ln 2 - 4$	ddM1	2.1
	$= 14(\ln 2)^2 - 12\ln 2 + 4$	A1	1.1b
		(5)	
			(8 marks)
	Notes		

(a)

B1: Correct strip width. May be implied by $\frac{1}{2}\times\frac{1}{2}\{....\}$ or $\frac{1}{4}\times\{....\}$

M1: Correct application of the trapezium rule.

Look for $\frac{1}{2}\times"h"\{0.4805+1.9218+2(0.8396+1.2069+1.5694)\}$ condoning slips in the digits.

The bracketing must be correct but it is implied by awrt 2.41

A1: 2.41 only. This is not awrt

(b)

M1: Attempts parts the correct way round to achieve $\alpha x(\ln x)^2-\beta\int \ln x \, dx$ o.e.

May be unsimplified (see scheme).

Watch for candidates who know or learn $\int \ln x \, dx = x\ln x - x$

who may write $\int(\ln x)^2 \, dx = \int(\ln x)(\ln x) \, dx = \ln x(x\ln x - x) - \int \frac{x\ln x - x}{x} \, dx$

A1: Correct expression which may be unsimplified

dM1: Attempts parts again to (only condone coefficient errors) to achieve $\alpha x(\ln x)^2-\beta x\ln x\pm\gamma x$ o.e.

ddM1: Applies the limits 4 and 2 to an expression of the form $\pm\alpha x(\ln x)^2\pm\beta x\ln x\pm\gamma x$, subtracts and applies $\ln 4 = 2\ln 2$ at least once. Both M's must have been awarded

A1: Correct answer

It is possible to do $\int(\ln x)^2 \, dx$ via a substitution $u=\ln x$ but it is very similar.

M1 A1, dM1: $\int u^2 e^u \, du = u^2 e^u - \int 2u e^u \, du, = u^2 e^u - 2ue^u \pm 2e^u$

ddM1: Applies appropriate limits and uses $\ln 4 = 2\ln 2$ at least once to an expression of the form $u^2 e^u - \beta u e^u \pm \gamma e^u$ Both M's must have been awarded