## Questions

**Q1.**

Given that *a* is a positive constant and

$$\int_{a}^{2a} \frac{t+1}{t} \, dt = \ln 7$$

show that *a* = ln*k*, where *k* is a constant to be found.

**(4)**

**(Total for question = 4 marks)**

**Q2.**

Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)} dx$ is independent of *k*,

**(4)**

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2} dx$ is inversely proportional to *k*.

**(3)**

**(Total for question = 7 marks)**

**Q3.**

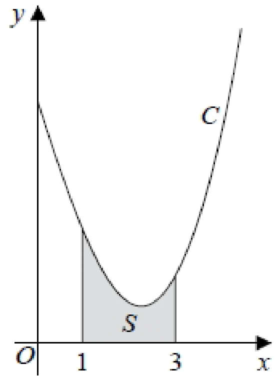


**Figure 4**

Figure 4 shows a sketch of part of the curve *C* with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region *S*, shown shaded in Figure 4, is bounded by the curve *C*, the line with equation $x = 1$, the *x*-axis and the line with equation $x = 3$

The table below shows corresponding values of *x* and *y* with the values of *y* given to 4 decimal places as appropriate.

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|
| $y$ | 3 | 2.3041 | 1.9242 | 1.9089 | 2.2958 |

(a) Use the trapezium rule, with all the values of *y* in the table, to obtain an estimate for the area of *S*, giving your answer to 3 decimal places.

**(3)**

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of *S*.

**(1)**

(c) Show that the exact area of *S* can be written in the form $\frac{a}{b} + \ln c$, where *a*, *b* and *c* are integers to be found.

**(6)**

(*In part c*, solutions based entirely on graphical or numerical methods are not acceptable.)

**(Total for question = 10 marks)**

**Q4.**

Show that

$$\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x = \frac{32}{15}\left(2+\sqrt{2}\right)$$

**(7)**

**(Total for question = 7 marks)**

**Q5.**

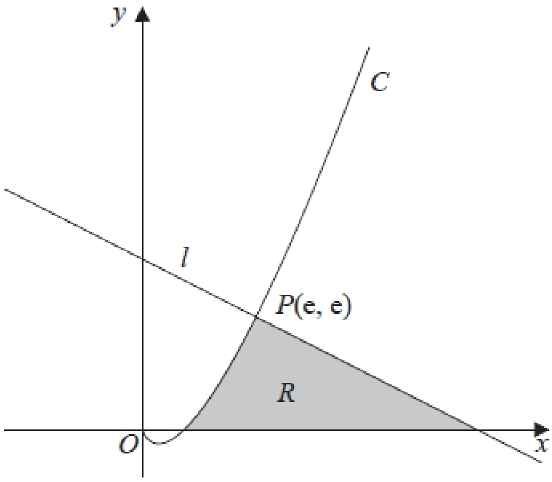


**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with equation $y = x \ln x, \quad x > 0$

The line $l$ is the normal to $C$ at the point $P(\mathrm{e}, \mathrm{e})$

The region $R$, shown shaded in Figure 2, is bounded by the curve $C$, the line $l$ and the $x$-axis.

Show that the exact area of $R$ is $A\mathrm{e}^2 + B$ where $A$ and $B$ are rational numbers to be found.

**(10)**

**(Total for question = 10 marks)**

**Q6.**

The curve $C$ with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \qquad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where $p$ and $q$ are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

**(3)**

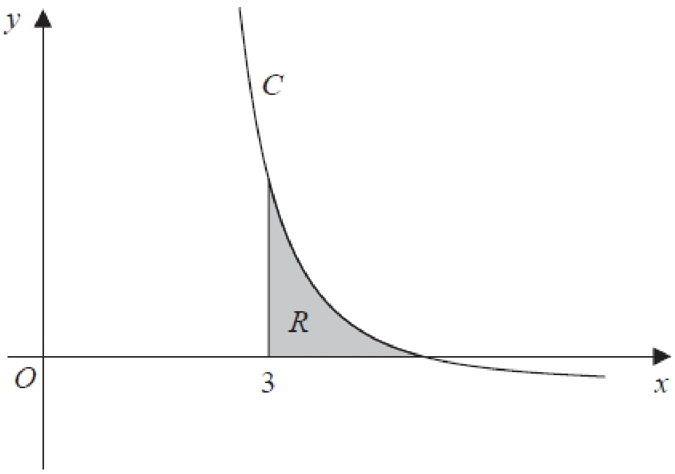


**Figure 4**

Figure 4 shows a sketch of part of the curve $C$. The region $R$, shown shaded in Figure 4, is bounded by the curve $C$, the $x$-axis and the line with equation $x = 3$

(b) Show that the exact value of the area of $R$ is $a\ln 2 + b\ln 3$, where $a$ and $b$ are rational constants to be found.

**(8)**

**(Total for question = 11 marks)**

**Q7.**

(a) Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,dx = \int_p^q \frac{2(u-1)^3}{u}\,du$$

where $p$ and $q$ are constants to be found.

**(3)**

(b) Hence show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,dx = A - B\ln 5$$

where $A$ and $B$ are constants to be found.

**(4)**

**(Total for question = 7 marks)**

## Mark Scheme

### Q1.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| | Writes $\int \frac{t+1}{t}\,\mathrm{d}t = \int 1+\frac{1}{t}\,\mathrm{d}t$ and attempts to integrate | M1 | 2.1 |
| | $= t+\ln t\ (+c)$ | M1 | 1.1b |
| | $(2a+\ln 2a)-(a+\ln a)=\ln 7$ | M1 | 1.1b |
| | $a=\ln\frac{7}{2}$ with $k=\frac{7}{2}$ | A1 | 1.1b |
| | | **(4 marks)** | |

**Notes:**

**M1:** Attempts to divide each term by $t$ or alternatively multiply each term by $t^{-1}$

**M1:** Integrates each term and knows $\int \frac{1}{t}\mathrm{d}t = \ln t$. The + $c$ is not required for this mark

**M1:** Substitutes in both limits, subtracts and sets equal to ln7

**A1:** Proceeds to $a=\ln\frac{7}{2}$ and states $k=\frac{7}{2}$ or exact equivalent such as 3.5

**Q2.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $\int \frac{2}{(3x-k)}\mathrm{d}x = \frac{2}{3}\ln(3x-k)$ | M1<br>A1 | 1.1a<br>1.1b |
| | $\int_k^{3k} \frac{2}{(3x-k)}\mathrm{d}x = \frac{2}{3}\ln(9k-k) - \frac{2}{3}\ln(3k-k)$ | dM1 | 1.1b |
| | $= \frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right) = \frac{2}{3}\ln 4$ oe | A1 | 2.1 |
| | | **(4)** | |
| (b) | $\int \frac{2}{(2x-k)^2}\mathrm{d}x = -\frac{1}{(2x-k)}$ | M1 | 1.1b |
| | $\int_k^{2k} \frac{2}{(2x-k)^2}\mathrm{d}x = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$ | dM1 | 1.1b |
| | $= \frac{2}{3k} \quad \left(\propto \frac{1}{k}\right)$ | A1 | 2.1 |
| | | **(3)** | |
| | | | **(7 marks)** |

**(a)**

**M1:** $\int \frac{2}{(3x-k)}\mathrm{d}x = A\ln(3x-k)$ Condone a missing bracket

**A1:** $\int \frac{2}{(3x-k)}\mathrm{d}x = \frac{2}{3}\ln(3x-k)$

Allow recovery from a missing bracket if in subsequent work $A\ln 9k - k \to A\ln 8k$

**dM1:** For substituting $k$ and $3k$ into their $A\ln(3x-k)$ and subtracting either way around

**A1:** Uses correct ln work and notation to show that I $= \frac{2}{3}\ln\left(\frac{8}{2}\right)$ or $\frac{2}{3}\ln 4$ oe (ie independent of $k$)

**(b)**

**M1:** $\int \frac{2}{(2x-k)^2}\mathrm{d}x = \frac{C}{(2x-k)}$

**dM1:** For substituting $k$ and $2k$ into their $\frac{C}{(2x-k)}$ and subtracting

**A1:** Shows that it is inversely proportional to $k$ Eg proceeds to the answer is of the form $A/k$ with $A = 2/3$

There is no need to perform the whole calculation. Accept from $-\frac{1}{(3k)} + \frac{1}{(k)} = \left(-\frac{1}{3}+1\right)\times\frac{1}{k} \propto \frac{1}{k}$

If the calculation is performed it must be correct.

**Do not isw here. They should know when they have an expression that is inversely proportional to $k$.**

You may see substitution used but the mark is scored for the same result. See below

$u = 2x - k \to \left[\frac{C}{u}\right]$ for M1 with limits $3k$ and $k$ used for dM1

**Q3.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Uses or implies $h = 0.5$ | B1 | 1.1b |
| | For correct form of the trapezium rule = | M1 | 1.1b |
| | $\frac{0.5}{2}\{3+2.2958+2(2.3041+1.9242+1.9089)\}=4.393$ | A1 | 1.1b |
| | | (3) | |
| (b) | Any valid statement reason, for example<br>• Increase the number of strips<br>• Decrease the width of the strips<br>• Use more trapezia | B1 | 2.4 |
| | | (1) | |
| (c) | For integration by parts on $\int x^2 \ln x \, dx$ | M1 | 2.1 |
| | $=\frac{x^3}{3}\ln x-\int\frac{x^2}{3}\,dx$ | A1 | 1.1b |
| | $\int -2x+5\,dx=-x^2+5x \quad (+c)$ | B1 | 1.1b |
| | All integration attempted and limits used<br>Area of $S=\int_1^3 \frac{x^2\ln x}{3}-2x+5\,dx=\left[\frac{x^3}{9}\ln x-\frac{x^3}{27}-x^2+5x\right]_{x=1}^{x=3}$ | M1 | 2.1 |
| | Uses correct ln laws, simplifies and writes in required form | M1 | 2.1 |
| | Area of $S=\frac{28}{27}+\ln 27 \quad (a=28, b=27, c=27)$ | A1 | 1.1b |
| | | (6) | |
| | | | **(10 marks)** |

**Notes:**

**(a)**

**B1:** States or uses the strip width $h = 0.5$. This can be implied by the sight of $\frac{0.5}{2}\{...\}$ in the trapezium rule

**M1:** For the correct form of the bracket in the trapezium rule. Must be $y$ values rather than $x$ values $\{\text{first } y \text{ value} + \text{last } y \text{ value} + 2\times(\text{sum of other } y \text{ values})\}$

**A1:** 4.393

**(b)**

**B1:** See scheme

**(c)**

**M1:** Uses integration by parts the right way around.

Look for $\int x^2 \ln x \, dx = Ax^3 \ln x - \int Bx^2 \, dx$

**A1:** $\frac{x^3}{3}\ln x - \int \frac{x^2}{3} \, dx$

**B1:** Integrates the $-2x+5$ term correctly $= -x^2 + 5x$

**M1:** All integration completed and limits used

**M1:** Simplifies using ln law(s) to a form $\frac{a}{b} + \ln c$

**A1:** Correct answer only $\frac{28}{27} + \ln 27$

**Q4.**

| Question | Scheme for Substitution | | Marks | AOs |
|---|---|---|---|---|
| | Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$<br>Award for<br>• Using a valid substitution $u =$..., changing the terms to $u$'s<br>• integrating and using appropriate limts . | | M1 | 3.1a |
| | Substitution<br>$u=\sqrt{x+2} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=2u$ oe | Substitution<br>$u=x+2 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=1$ oe | B1 | 1.1b |
| | $\int 2x\sqrt{x+2}\,\mathrm{d}x$<br>$=\int A(u^2\pm 2)u^2\mathrm{d}u$ | $\int 2x\sqrt{x+2}\,\mathrm{d}x$<br>$=\int A(u\pm 2)\sqrt{u}\mathrm{d}u$ | M1 | 1.1b |
| | $=Pu^5 \pm Qu^3$ | $=Su^{\frac{5}{2}} \pm Tu^{\frac{3}{2}}$ | dM1 | 2.1 |
| | $=\frac{4}{5}u^5-\frac{8}{3}u^3$ | $=\frac{4}{5}u^{\frac{5}{2}}-\frac{8}{3}u^{\frac{3}{2}}$ | A1 | 1.1b |
| | Uses limits 2 and $\sqrt{2}$ the correct way around | Uses limits 4 and 2 the correct way around | ddM1 | 1.1b |
| | $=\frac{32}{15}\left(2+\sqrt{2}\right)$ * | | A1* | 2.1 |
| | | | (7) | |
| | | | | **(7 marks)** |

**M1:** For attempting to integrate using substitution. Look for

- terms and limits changed to $u$'s. Condone slips and errors/omissions on changing $\mathrm{d}x \rightarrow \mathrm{d}u$
- attempted multiplication of terms and raising of at least one power of $u$ by one. Condone slips
- Use of at least the top correct limit. For instance if they go back to $x$'s the limit is 2

**B1:** For substitution it is for giving the substitution and stating a correct $\frac{\mathrm{d}x}{\mathrm{d}u}$

Eg, $u=\sqrt{x+2} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=2u$ or equivalent such as $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2\sqrt{x+2}}$

**M1:** It is for attempting to get all aspects of the integral in terms of '$u$'.
All terms must be attempted including the d$x$. You are only condoning slips on signs and coefficients

**dM1:** It is for using a correct method of expanding and integrating each term (seen at least once) . It is dependent upon the previous M

**A1:** Correct answer in $x$ or $u$ See scheme

**ddM1:** Dependent upon the previous M, it is for using the correct limits for their integral, **the correct way around**

**A1*:** Proceeds correctly to $=\frac{32}{15}\left(2+\sqrt{2}\right)$. **Note that this is a given answer**

There must be at one least correct intermediate line between $\left[\frac{4}{5}u^5-\frac{8}{3}u^3\right]_{\sqrt{2}}^{2}$ and $\frac{32}{15}\left(2+\sqrt{2}\right)$

| Question Alt | Scheme for by parts | Marks | AOs |
|---|---|---|---|
| | Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,dx$<br>Award for<br>• using by parts the correct way around<br>• and using limits | M1 | 3.1a |
| | $\int\left(\sqrt{x+2}\right)dx = \frac{2}{3}(x+2)^{\frac{3}{2}}$ | B1 | 1.1b |
| | $\int 2x\sqrt{x+2}\,dx = Ax(x+2)^{\frac{3}{2}} - \int B(x+2)^{\frac{3}{2}}(dx)$ | M1 | 1.1b |
| | $= Ax(x+2)^{\frac{3}{2}} - C(x+2)^{\frac{5}{2}}$ | dM1 | 2.1 |
| | $= \frac{4}{3}x(x+2)^{\frac{3}{2}} - \frac{8}{15}(x+2)^{\frac{5}{2}}$ | A1 | 1.1b |
| | Uses limits 2 and 0 the correct way around | ddM1 | 1.1b |
| | $= \frac{32}{15}\left(2+\sqrt{2}\right)$ | A1* | 2.1 |
| | | (7) | |

**M1:** For attempting using by parts to solve It is a problem- solving mark and all elements do not have to be correct.

- the formula applied the correct way around. You may condone incorrect attempts at integrating $\sqrt{x+2}$ for this problem solving mark
- further integration, again, this may not be correct, and the use of at least the top limit of 2

**B1:** For $\int\left(\sqrt{x+2}\right)dx = \frac{2}{3}(x+2)^{\frac{3}{2}}$ oe May be awarded $\int_0^2 2x\sqrt{x+2}\,dx \rightarrow x^2 \times \frac{2(x+2)^{\frac{3}{2}}}{3}$

**M1:** For integration by parts the right way around. Award for $Ax(x+2)^{\frac{3}{2}} - \int B(x+2)^{\frac{3}{2}}(dx)$

**dM1:** For integrating a second time. Award for $Ax(x+2)^{\frac{3}{2}} - C(x+2)^{\frac{5}{2}}$

**A1:** $\frac{4}{3}x(x+2)^{\frac{3}{2}} - \frac{8}{15}(x+2)^{\frac{5}{2}}$ which may be un simplified

**ddM1:** Dependent upon the previous M, it is for using the limits 2 and 0 the **correct way around**

**A1*:** Proceeds to $= \frac{32}{15}\left(2+\sqrt{2}\right)$. **Note that this is a given answer.**

At least one correct intermediate line must be seen. (See substitution). You would condone missing dx's

**Q5.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| | $C: y = x\ln x$; $l$ is a normal to $C$ at $P(\mathrm{e}, \mathrm{e})$<br>Let $x_A$ be the $x$-coordinate of where $l$ cuts the $x$-axis | | |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\frac{1}{x}\right) \quad \{= 1 + \ln x\}$ | M1<br>A1 | 2.1<br>1.1b |
| | $x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\frac{1}{2} \Rightarrow y - \mathrm{e} = -\frac{1}{2}(x - \mathrm{e})$<br>$y = 0 \Rightarrow -\mathrm{e} = -\frac{1}{2}(x - \mathrm{e}) \Rightarrow x = \ldots$ | M1 | 3.1a |
| | $l$ meets $x$-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln \mathrm{e}$) | A1 | 1.1b |
| | {Areas:} **either** $\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$ **or** $\frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ | M1 | 2.1 |
| | $\left\{\int x\ln x\,\mathrm{d}x = \right\}\ \frac{1}{2}x^2\ln x - \int \frac{1}{x}.\left(\frac{x^2}{2}\right)\{\mathrm{d}x\}$ | M1 | 2.1 |
| | $\left\{= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\ \{\mathrm{d}x\}\right\} = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2$ | dM1<br>A1 | 1.1b<br>1.1b |
| | $\text{Area}(R_1) = \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$ ; $\text{Area}(R_2) = \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$<br>and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2)$ $\{= \frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2\}$ | M1 | 3.1a |
| | $\text{Area}(R) = \frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$ | A1 | 1.1b |
| | | (10) | |

| Notes for Question | |
|---|---|
| **M1:** | Differentiates by using the product rule to give $\ln x + x(\text{their } g'(x))$, where $g(x) = \ln x$ |
| **A1:** | Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified |
| **M1:** | Complete strategy to find the $x$ coordinate where their normal to $C$ at $P(\text{e}, \text{e})$ meets the $x$-axis<br>i.e. Sets $y=0$ in $y - \text{e} = m_N(x - \text{e})$ to find $x = \ldots$ |
| **Note:** | $m_T$ is found by using calculus and $m_N \neq m_T$ |
| **A1:** | $l$ meets $x$-axis at $x = 3\text{e}$, allowing un-simplified values for $x$ such as $x = 2\text{e} + \text{e}\ln\text{e}$ |
| **Note:** | Allow $x =$ awrt 8.15 |
| **M1:** | Scored for either<br>• Area under curve $= \int_1^{\text{e}} x\ln x \, dx = [\ \ldots\ ]_1^{\text{e}} = \ldots$, with limits of e and 1 and some attempt to substitute these and subtract<br>• or Area under line $= \frac{1}{2}((\text{their } x_A) - \text{e})\text{e}$, with a valid attempt to find $x_A$ |
| **M1:** | Integration by parts the correct way around to give $Ax^2\ln x - \int B\left(\frac{x^2}{x}\right)\{dx\}$; $A \neq 0, B > 0$ |
| **dM1:** | **dependent on the previous M mark**<br>Integrates the second term to give $\pm\lambda x^2$; $\lambda \neq 0$ |
| **A1:** | $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2$ |
| **M1:** | Complete strategy of finding the area of $R$ by finding the sum of two key areas. See scheme. |
| **A1:** | $\frac{5}{4}\text{e}^2 + \frac{1}{4}$ |
| **Note:** | $\text{Area}(R_2)$ can also be found by integrating the line $l$ between limits of e and their $x_A$<br>i.e. $\text{Area}(R_2) = \int_{\text{e}}^{\text{their } x_A} \left(-\frac{1}{2}x + \frac{3}{2}\text{e}\right) dx = [\ \ldots\ ]_{\text{e}}^{\text{their } x_A} = \ldots$ |
| **Note:** | <u>**Calculator approach with no algebra, differentiation or integration seen:**</u><br>• Finding $l$ cuts through the $x$-axis at awrt 8.15 is 2nd M1 2nd A1<br>• Finding area between curve and the $x$-axis between $x = 1$ and $x = \text{e}$ to give awrt 2.10 is 3rd M1<br>• Using the above information (must be seen) to apply $\text{Area}(R) = 2.0972\ldots + 7.3890\ldots = 9.4862\ldots$ is final M1<br>Therefore, a maximum of 4 marks out of the 10 available. |

**Q6.**

| Part | Working or answer an examiner might expect to see | Mark | Notes |
|---|---|---|---|
| (a) | The asymptote is found where $2x - q = 0$<br>Hence $q = 4$ | B1 | This mark is given for explaining that the asymptote at $x = 2$ is a solution of $2x - q = 0$ |
| | $y = \frac{p-3x}{(2x-4)(x+3)}$<br>$\frac{1}{2} = \frac{p-9}{(6-4)(3+3)}$ | M1 | This mark is given for substituting $x = 3$, $y = \frac{1}{2}$ (and $q = 4$) |

| | | | |
|---|---|---|---|
| (b) | $\frac{15-3x}{(2x-4)(x+3)} = \frac{A}{(2x-4)} + \frac{B}{(x+3)}$ | M1 | This mark is given for a method to use partial fractions |
| | $= \frac{1.8}{(2x-4)} - \frac{2.4}{(x+3)}$ | M1 | This mark is given for finding values for $A$ and $B$ |
| | $= \frac{0.9}{(x-2)} - \frac{2.4}{(x+3)}$ | A1 | This mark is given for a fully simplified expression |
| | $I = \int \frac{15-3x}{(2x-4)(x+3)}\,dx$<br>$= m \ln(2x-4) + n \ln(x+3)$ | M1 | This mark is given for a method to integrate to find the area of $R$ |
| | $= 0.9 \ln(2x-4) + 2.4 \ln(x+3)$ | A1 | This mark is given for a correct expression for the area of $R$ |
| | Area $R = \left[0.9\ln(2x-4) - 2.4\ln(x+3)\right]_3^5$ | M1 | This mark is given for deducing an expression for the area of $R$<br>($y = 0$ when $x = 5$) |
| | $= [0.9 \ln 6 - 2.4 \ln 8] - [0.9 \ln 2 - 2.4 \ln 6]$<br>$= [0.9 \ln 6 + 2.4 \ln 6] - [7.2 \ln 2 + 0.9 \ln 2]$<br>$= 3.3 \ln 6 - 8.1 \ln 2$<br>$= 3.3 \ln 3 + 3.3 \ln 2 - 8.1 \ln 2$ | M1 | This mark is given for a method to find the exact area of $R$ |
| | $= 3.3 \ln 3 - 4.8 \ln 2$ | A1 | This mark is given for a correct value of the area of $R$ with $a = 3.3$ and $b = 4.8$ |

**Q7.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $u=1+\sqrt{x}\Rightarrow x=(u-1)^2\Rightarrow\dfrac{dx}{du}=2(u-1)$ or $u=1+\sqrt{x}\Rightarrow\dfrac{du}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}$ | B1 | 1.1b |
| | $\int\dfrac{x}{1+\sqrt{x}}dx=\int\dfrac{(u-1)^2}{u}2(u-1)du$ or $\int\dfrac{x}{1+\sqrt{x}}dx=\int\dfrac{x}{u}\times 2x^{\frac{1}{2}}du=\int\dfrac{2x^{\frac{3}{2}}}{u}du=\int\dfrac{2(u-1)^3}{u}du$ | M1 | 2.1 |
| | $\int_0^{16}\dfrac{x}{1+\sqrt{x}}dx=\int_1^5\dfrac{2(u-1)^3}{u}du$ | A1 | 1.1b |
| | | (3) | |
| (b) | $2\int\dfrac{u^3-3u^2+3u-1}{u}du=2\int\left(u^2-3u+3-\dfrac{1}{u}\right)du=...$ | M1 | 3.1a |
| | $=(2)\left[\dfrac{u^3}{3}-\dfrac{3u^2}{2}+3u-\ln u\right]$ | A1 | 1.1b |
| | $=2\left[\dfrac{5^3}{3}-\dfrac{3(5)^2}{2}+3(5)-\ln 5-\left(\dfrac{1}{3}-\dfrac{3}{2}+3-\ln 1\right)\right]$ | dM1 | 2.1 |
| | $=\dfrac{104}{3}-2\ln 5$ | A1 | 1.1b |
| | | (4) | |
| | | | (7 marks) |

| Notes |
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| (a)<br>B1: Correct expression for $\frac{dx}{du}$ or $\frac{du}{dx}$ (or $u'$) or d$x$ in terms of d$u$ or d$u$ in terms of d$x$<br>M1: Complete method using the given substitution.<br>This needs to be a correct method for their $\frac{dx}{du}$ or $\frac{du}{dx}$ leading to an integral in terms of $u$ only (ignore any limits if present) so for each case you need to see:<br>$\frac{dx}{du}=f(u) \to \int \frac{x}{1+\sqrt{x}}dx = \int \frac{(u-1)^2}{u} f(u)du$<br>$\frac{du}{dx}=g(x) \to \int \frac{x}{1+\sqrt{x}}dx = \int \frac{x}{u}\times\frac{du}{g(x)} = \int h(u)du$. In this case you can condone slips with coefficients e.g. allow $\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}} \to \int \frac{x}{1+\sqrt{x}}dx = \int \frac{x}{u}\times\frac{x^{\frac{1}{2}}}{2}du = \int h(u)du$<br>but not $\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}} \to \int \frac{x}{1+\sqrt{x}}dx = \int \frac{x}{u}\times\frac{x^{-\frac{1}{2}}}{2}du = \int h(u)du$<br>A1: All correct with correct limits and no errors. The "d$u$" must be present but may have been omitted along the way but it must have been seen at least once before the final answer. The limits can be seen as part of the integral or stated separately.<br>(b)<br>M1: Realises the requirement to cube the bracket and divide through by $u$ and makes progress with the integration to obtain at least 3 terms of the required form e.g. 3 from $ku^3$, $ku^2$, $ku$, $k\ln u$<br>A1: Correct integration. This mark can be scored with the "2" still outside the integral or even if it has been omitted. But if the "2" has been combined with the integrand, the integration must be correct.<br>dM1: Completes the process by applying their "changed" limits and subtracts the right way round **Depends on the first method mark.**<br>A1: Cao (Allow equivalents for $\frac{104}{3}$ e.g. $\frac{208}{6}$) |