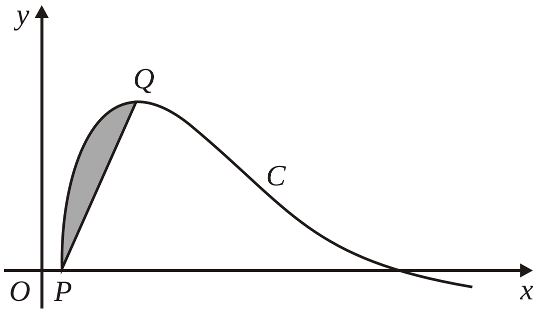
1.



The figure above shows a sketch of part of the curve C with equation

$$y = \sin(\ln x), \qquad x \geq 1.$$

The point Q, on C, is a maximum.

(a) Show that the point $P(1, 0)$ lies on C.

(1)

(b) Find the coordinates of the point Q.

(5)

(c) Find the area of the shaded region between C and the line PQ.

(9)
(Total 15 marks)

2. The function f is defined by

$$\text{f}: x \mapsto 3 + 2e^x, \quad x \in \mathbb{R}.$$

(a) Evaluate $\int_0^1 f(x)\,dx$, giving your answer in terms of e.

(3)

The curve C, with equation $y = \text{f}(x)$, passes through the y-axis at the point A. The tangent to C at A meets the x-axis at the point $(c, 0)$.

(b) Find the value of c.

(4)

The function g is defined by

$$g: x \mapsto \frac{5x+2}{x+4}, \quad x \in \mathbb{R}, \quad x > -4.$$

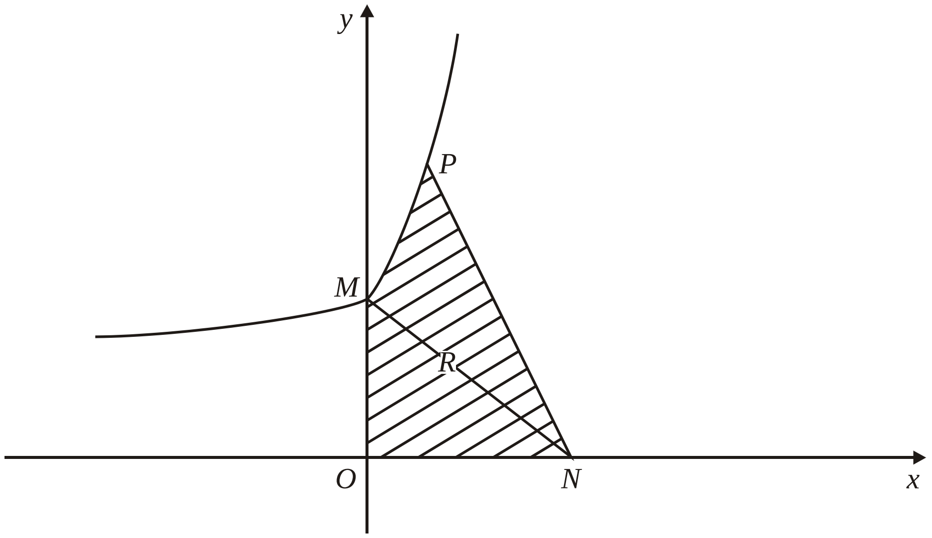
(c) Find an expression for $g^{-1}(x)$.

(3)

(d) Find gf(0).

(2)
(Total 12 marks)

3.



The curve C with equation $y = 2e^x + 5$ meets the y-axis at the point M, as shown in the diagram above.

(a) Find the equation of the normal to C at M in the form $ax + by = c$, where a, b and c are integers.

(4)

This normal to C at M crosses the x-axis at the point $N(n, 0)$.

(b) Show that $n = 14$.

(1)

The point $P(\ln 4, 13)$ lies on C. The finite region R is bounded by C, the axes and the line PN, as shown in the diagram above.

(c) Find the area of R, giving your answer in the form $p + q \ln 2$, where p and q are integers to be found.

(7)
(Total 12 marks)

1. (a) $x = 1$; $y = \sin(\ln 1) = \sin 0 = 0$

$\therefore$ P = (1, 0) and P lies on C B1 c.s.o. 1

(b) $y' = \frac{1}{x}\cos(\ln x)$ A1

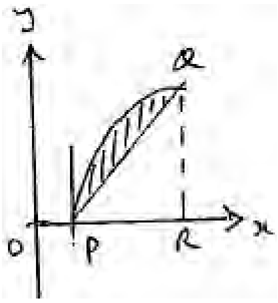
$y' = 0$ at Q $\therefore \cos(\ln x) = 0$ $\therefore \ln x = \frac{\pi}{2}$

$x = e^{\frac{\pi}{2}}$

$\therefore Q = \left(e^{\frac{\pi}{2}}, \sin(\ln e^{\frac{\pi}{2}})\right)$ A1

$= (e^{\frac{\pi}{2}}, 1)$ A1 5

(c)



Area = $\int_1^{e^{\frac{\pi}{2}}} \sin(\ln x)\mathrm{d}x$ – Area Δ PQR (correct approach)

Area Δ PQR = $\frac{1}{2} \times 1 \times (e^{\frac{\pi}{2}} - 1)$ B1

for integral; let $\ln x = u$ $\therefore x = e^u$ (substitution)

$\frac{1}{x}dx = du$ $\therefore dx = e^u du$

$\underline{\text{F}} = \int_0^{\frac{\pi}{2}} \sin u.(e^u du)$ A1

$= \left[e^u \sin u\right]_0^{\frac{\pi}{2}} - \int e^u \cos u du$

$= e^{\frac{\pi}{2}} - \left[e^u \cos u\right]_0^{\frac{\pi}{2}} - \int e^u \sin u du$

$\therefore 2\text{I} = e^{\frac{\pi}{2}} + 1$

$\text{I} = \frac{1}{2}(1 + e^{\frac{\pi}{2}}) = 1$ (I) A1

$\therefore \text{Area} = \frac{1}{2}(1 + e^{\frac{\pi}{2}}) - \frac{1}{2}(-1 + e^{\frac{\pi}{2}}) = 1$ A1

[9]

2. (a) I = $3x + 2e^x$ B1

Using limits correctly to give 1 + 2e. (c.a.o.) A1 3

must subst 0 and 1 and subtract

(b) $A = (0, 5)$; B1

$y = 5$

$\frac{dy}{dx} = 2e^x$ B1

Equation of tangent: $y = 2x + 5$; c = –2.5 A1 4

attempting to find eq. of tangent and subst in $y = 0$,
must be linear equation

(c) $y = \frac{5x+2}{x+4} \Rightarrow yx + 4y = 5x + 2 \Rightarrow 4y - 2 = 5x - xy$ A1

putting y = and att. to rearrange to find x .

$g^{-1}(x) = \frac{4x-2}{5-x}$ or equivalent A1 3

must be in terms of x

(d) gf(0) = g(5); =3 A1 2

att to put 0 into f and then their answer into g

[12]

3. (a) M is (0, 7) B1

$\frac{dy}{dx} = 2e^x$

Attempt $\frac{dy}{dx}$

$\therefore$ gradient of normal is $-\frac{1}{2}$

ft their $y'(0)$ *or* $= -\frac{1}{2}$

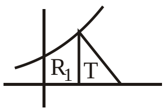
(Must be a number)

$\therefore$ equation of normal is $y - 7 = -\frac{1}{2}(x - 0)$ or $\underline{x + 2y - 14 = 0}$

$\underline{x + 2y = 14}$ o.e. A1 4

(b) $y = 0$, $x = 14$ $\therefore$ $\underline{N \text{ is } (14, 0)}$ (*) B1 cso 1

(c)



$\int (2e^x + 5)\,dx = [2e^x + 5x]$

some correct $\int$

$R_1 = \int_0^{\ln 4} (2e^x + 5)\,dx = (2 \times 4 + 5 \ln 4) - (2 + 0)$

limits used

$= 6 + 5 \ln 4$ A1

$T = \frac{1}{2} \times 13 \times (14 - \ln 4)$ B1

Area of T

$T = 13(7 - \ln 2)$; $R_1 = 6 + 10 \ln 2$ B1

Use of ln 4 = 2ln 2

$R = T + R_1$, $\underline{R = 97 - 3 \ln 2}$ A1 7

[12]

1. This was the question in which many candidates earned their highest marks. It was also the one for which most S marks were gained. Virtually all candidates scored the first mark. Differentiation was generally good in part (b) and many candidates scored all 5 of these marks. A common error was to state that ln x=1. There were also many good attempts at part (c). Nearly all recognized the need to take the difference of two areas. Those who sought to find the area of the triangle by forming the equation of the line and then integrating usually came unstuck in a mass of algebra and they rarely obtained the correct value. Fortunately most simply used half the base x height! Integration of y was usually well done. Similar numbers of candidates used direct integration by parts ($x$sin(ln x) etc.) as used the substitution u=ln x, resulting ine^u sin u du. Many were able to complete the two cycles of parts and obtain the correct answer.

2. This question was relatively well answered. In part (a) most candidates integrated correctly though some candidates did not use the limit $x = 0$. In part (b) a minority of candidates left the gradient of the tangent as $2e^{2x}$. A few thought the gradient of the tangent was -1/2. In part (c) the majority of candidates knew what to do but poor algebraic skills gave incorrect answers Part (d) was usually well done, though some found $g^{-1}f(0)$ and the inevitable few fg(0).

3. Whilst the majority of answers to part (a) were fully correct, some candidates found difficulties here. A small number failed to find the coordinates of M correctly with (0, 5) being a common mistake. Others knew the rule for perpendicular gradients but did not appreciate that the gradient of a normal must be numerical. A few students did not show clearly that the gradient of the curve at $x = 0$ was found from the derivative, they seemed to treat $y = 2e^x + 5$ and assumed the gradient was always 2. Some candidates failed to obtain the final mark in this section because they did not observe the instruction that a, b and c must be integers.

For most candidates part (b) followed directly from their normal equation. It was disappointing that those who had made errors in part (a) did not use the absence of $n = 14$ here as a pointer to check their working in the previous part. Most preferred to invent all sorts of spurious reasons to justify the statement.

Many candidates set out a correct strategy for finding the area in part (c). The integration of the curve was usually correct but some simply ignored the lower limit of 0. Those who used the simple “half base times height” formula for the area of the triangle, and resisted the lure of their calculator, were usually able to complete the question. Some tried to find the equation of PN and integrate this but they usually made no further progress. The demand for exact answers proved more of a challenge here than in 6(c) but many candidates saw clearly how to simplify $2e^{\ln 4}$ and convert ln 4 into 2 ln 2 on their way to presenting a fully correct solution.