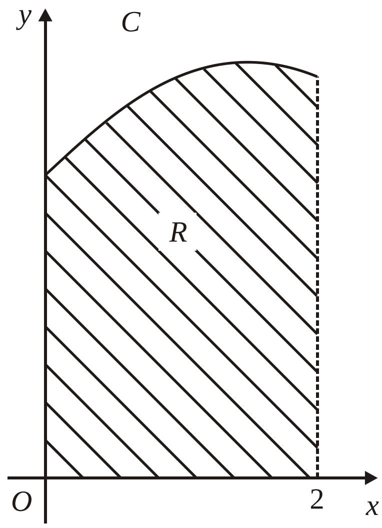
1. Find, in terms of e, the exact value of $\int_1^{e}\left(1+\frac{5}{x}\right)\,dx$.

(Total 4 marks)

2.



The curve C has equation $y = f(x)$, $x \in \mathbb{R}$. The diagram above shows the part of C for which $0 \le x \le 2$.

Given that

$$\frac{dy}{dx} = e^x - 2x^2,$$

and that C has a single maximum, at $x = k$,

(a) show that $1.48 < k < 1.49$.

(3)

Given also that the point (0, 5) lies on C,

(b) find $f(x)$.

(4)

The finite region R is bounded by C, the coordinate axes and the line $x = 2$.

(c) Use integration to find the exact area of R.

(4)
(Total 11 marks)

1. $\int \left(1 + \frac{5}{x}\right) \mathrm{d}x = x + 5\ln x$ A1

$\left[x + 5\ln x\right]_1^{\mathrm{e}}$ = (e + 5) – 1 = e + 4 Correct use of limits A1 4

[4]

2. (a) $\mathrm{f}^{-1}(x) = 0$ for maximum (or stationary point or turning point) B1

$\mathrm{f}^1\,(1.48) = e^{1.48} - 2 \times 1.48^2 = 0.0121\ldots$

$f^1\,(1.49) = \qquad = -0.0031\ldots$

change of sign $\therefore$ root / maximum in range A1 3

May be $\Rightarrow$ if maximum mentioned at A1

One value correct to 1 S.F.

A1 Both correct and comment

(b) $y = e^x - \frac{2}{3}x^3\ (+c)$ A1

at (0, 5) $5 = e^0 - 0 + c$

$\underline{c = 4}\ \left(y = e^x - \frac{2}{3}x^3 + 4\right)\ (c = 4)$ 4

Some correct $\int$

A1 $e^x - \frac{2}{3}x^3$

Attempt to use (0,5)

No + c is M0

(c) Area = $\int_0^2 \left(e^x - \frac{2}{3}x^3 + 4\right)\mathrm{d}x$

$= \left[e^x - \frac{2}{12}x^4 + 4x\right]_0^2$ A1ft

$= \left(e^2 - \frac{16}{6} + 8\right) - (\mathrm{e}^{\ddot{o}} - 0 + 0)$

$= \underline{e^2 + 4\frac{1}{3}}$ or $\underline{e^2 + \frac{13}{3}}$ A1 cao 4

Some correct $\int$ *<u>other</u> than* $e^x \to e^x$.

A1 ft [ ] ft their c($\neq$0).

Attempt both limits

[11]

1. This question was generally well done, although many lost the final mark by leaving their answer as $e + 5\ln e - 1$, instead of tidying up to e + 4.

2. Most candidates were familiar with the type of question in part (a) but a few still failed to evaluate the derivative at 1.48 and 1.49.

Simply stating that f′(1.48) > 0 and f′(1.49) < 0 is not sufficient. Some candidates failed to appreciate the answer their calculator gave them was in standard form and –3.1 instead of –0.0031 was a common mistake. In part (b) most integrated successfully but some forgot to include the constant of integration and were not then able to use the point (0, 5) properly. There were still a few who substituted $\frac{dy}{dx}$ into $y - y_1 = m(x - x_1)$.

The technique required in part (c) was well known but many candidates failed to heed the instruction to give the exact area.