INTEGRATION — Answers

1 **a** at A, $x = 0$ $\therefore$ $A\ (0, 4)$

at B, $y = 0$

$(x^{\frac{1}{2}} - 2)^2 = 0$

$x^{\frac{1}{2}} = 2$

$x = 4$ $\therefore$ $B\ (4, 0)$

b $= \int_0^4 (x - 4x^{\frac{1}{2}} + 4)\ dx$

$= [\frac{1}{2}x^2 - \frac{8}{3}x^{\frac{3}{2}} + 4x]_0^4$

$= (8 - \frac{64}{3} + 16) - 0$

$= \frac{8}{3}$

2 $= \int_1^2 (\frac{3}{2}x + \frac{1}{2}x^{-2})\ dx$

$= [\frac{3}{4}x^2 - \frac{1}{2}x^{-1}]_1^2$

$= (3 - \frac{1}{4}) - (\frac{3}{4} - \frac{1}{2})$

$= \frac{5}{2}$

3 **a** $4^{x+1} = 32$

$(2^2)^{x+1} = 2^5$

$2x + 2 = 5$

$x = \frac{3}{2}$

b

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$
4^{x+1}	4	8	16	32

$\therefore$ area $\approx \frac{1}{2} \times \frac{1}{2} \times [4 + 32 + 2(8 + 16)]$

$= 21$

4 **a** at A, $x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$ (at O) or 2 $\therefore$ $A\ (2, 0)$

at B, $x^2 - 2x = x$

$x(x - 3) = 0$

$x = 0$ (at O) or 3 $\therefore$ $B\ (3, 3)$

b $\int_0^2 (x^2 - 2x)\ dx$

$= [\frac{1}{3}x^3 - x^2]_0^2$

$= (\frac{8}{3} - 4) - 0 = -\frac{4}{3}$

$\therefore$ area $= \frac{4}{3}$

c area below curve between A and B

$= \int_2^3 (x^2 - 2x)\ dx$

$= [\frac{1}{3}x^3 - x^2]_2^3$

$= (9 - 9) - (-\frac{4}{3}) = \frac{4}{3}$

area below straight line OB

$= \frac{1}{2} \times 3 \times 3 = \frac{9}{2}$

area between curve and line

$= \frac{9}{2} - \frac{4}{3} + \frac{4}{3}$

$= \frac{9}{2}$

5 **a**

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.319	1.024	0

b $\approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 0 + 2(1.319 + 1.024)]$

$= 1.49$ (3sf)

c under-estimate

curve passes above top of each trapezium

6 $\int_1^k (3 - 4x^{-2})\ dx$

$= [3x + 4x^{-1}]_1^k$

$= (3k + \frac{4}{k}) - (3 + 4)$

$\therefore\ 3k + \frac{4}{k} - 7 = 6$

$3k^2 - 13k + 4 = 0$

$(3k - 1)(k - 4) = 0$

$k > 1\ \therefore\ k = 4$

7 **a** $\frac{dy}{dx} = 3x^2 - 6x$

SP: $3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ (at P) or 2

$\therefore\ Q\ (2, 1)$

b $x^3 - 3x^2 + 5 = 5$

$x^2(x - 3) = 0$

$x = 0$ (at P) or 3

$\therefore\ R\ (3, 5)$

c area below curve

$= \int_0^3 (x^3 - 3x^2 + 5)\ dx$

$= [\frac{1}{4}x^4 - x^3 + 5x]_0^3$

$= (\frac{81}{4} - 27 + 15) - 0 = \frac{33}{4}$

area below line

$= 3 \times 5 = 15$

shaded area

$= 15 - \frac{33}{4}$

$= 6\frac{3}{4}$

8 **a** (2, 0)

b

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$(2 - x)^3$	8	$\frac{27}{8}$	1	$\frac{1}{8}$	0

area $\approx \frac{1}{2} \times \frac{1}{2} \times [8 + 0 + 2(\frac{27}{8} + 1 + \frac{1}{8})]$

$= 4\frac{1}{4}$

c $= 2^3 + 3(2^2)(-x) + 3(2)(-x)^2 + (-x)^3$

$= 8 - 12x + 6x^2 - x^3$

d area $= \int_0^2 (8 - 12x + 6x^2 - x^3)\ dx$

$= [8x - 6x^2 + 2x^3 - \frac{1}{4}x^4]_0^2$

$= (16 - 24 + 16 - 4) - 0$

$= 4$

$\therefore$ % error $= \frac{4\frac{1}{4} - 4}{4} \times 100\% = 6.25\%$