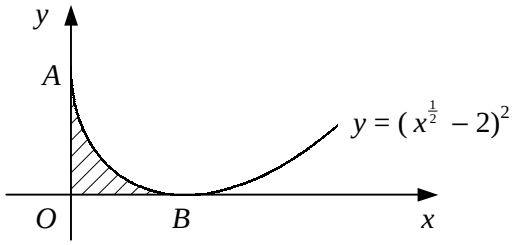
INTEGRATION

1



The diagram shows the curve with the equation $y = (x^{\frac{1}{2}} - 2)^2$. The curve meets the y-axis at the point A and the x-axis at the point B.

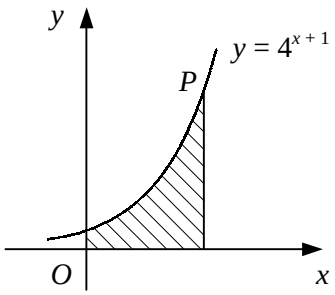
a Find the coordinates of the points A and B. **(3)**

b Find the area of the shaded region enclosed by the curve and the coordinate axes. **(6)**

2 Evaluate

$$\int_1^2 \frac{3x^3 + 1}{2x^2}\, dx.$$ **(5)**

3



The diagram shows the curve with equation $y = 4^{x+1}$.

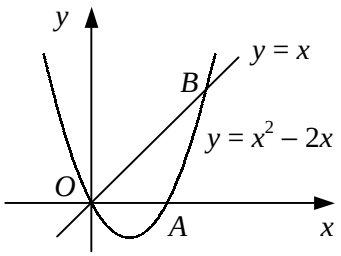
The point P on the curve has y-coordinate 32.

a Find the x-coordinate of P. **(3)**

The shaded region is bounded by the curve, the coordinate axes and the line through P parallel to the y-axis.

b Use the trapezium rule with 4 equally-spaced ordinates to estimate the area of the shaded region. **(5)**

4



The diagram shows the curve $y = x^2 - 2x$ and the line $y = x$. The curve crosses the x-axis at the origin, O, and at the point A. The line intersects the curve at O and at the point B.

a Find the coordinates of the points A and B. **(4)**

b Find the area of the region enclosed by the curve and the x-axis. **(5)**

c Show that the area of the region enclosed by the curve and the line $y = x$ is $\frac{9}{2}$. **(5)**

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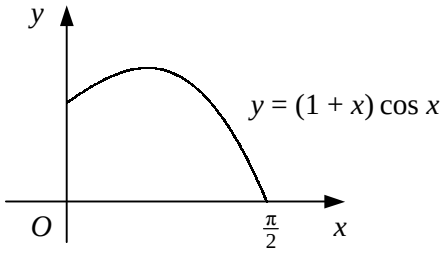
5



The diagram shows the curve with equation $y = (1 + x)\cos x$, $0 \leq x \leq \frac{\pi}{2}$.

a Copy and complete the table below for points on the curve, giving the y values correct to 3 decimal places where appropriate.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y				

(3)

b Use the trapezium rule with the values in your table to estimate the area of the region bounded by the curve and the coordinate axes. **(4)**

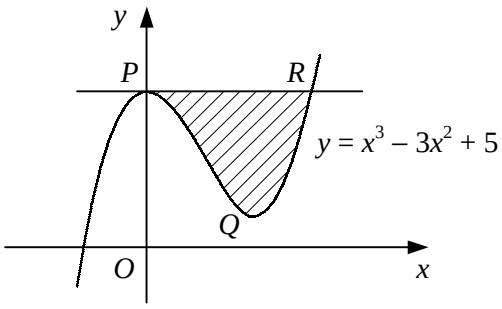
c State, with a reason, whether your answer to part **b** is an under-estimate or an over-estimate of the true area. **(2)**

6 Given that

$$\int_1^k \left(3 - \frac{4}{x^2}\right) \mathrm{d}x = 6,$$

and that $k > 1$, find the value of the constant k. **(7)**

7



The diagram shows the curve with the equation $y = x^3 - 3x^2 + 5$. The curve is stationary at the point P (0, 5) and at the point Q.

a Find the coordinates of the point Q. **(5)**

The straight line passing through the point P parallel to the x-axis intersects the curve again at the point R.

b Find the coordinates of the point R. **(2)**

c Find the area of the shaded region enclosed by the curve and the straight line PR. **(7)**

8 The finite region R is bounded by the curve $y = (2 - x)^3$ and the coordinate axes.

a State the coordinates of the point where the curve crosses the x-axis. **(1)**

b Use the trapezium rule with 4 intervals of equal width to estimate the area of R. **(5)**

c Expand $(2 - x)^3$ in ascending powers of x. **(2)**

d Hence, using integration, find the percentage error in the estimate for the area of R found in part **b**. **(6)**

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