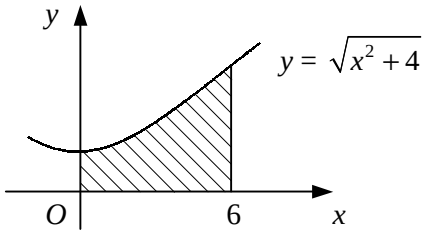
INTEGRATION

1 Evaluate

a $\int_1^4 \frac{2}{x^2}\,\mathrm{d}x$, **(3)**

b $\int_0^2 (x-3)^2\,\mathrm{d}x$. **(4)**

2



The shaded region in the diagram is bounded by the curve $y = \sqrt{x^2+4}$, the x-axis and the lines $x = 0$ and $x = 6$.

a Use the trapezium rule with three intervals of equal width to estimate the area of the shaded region. **(5)**

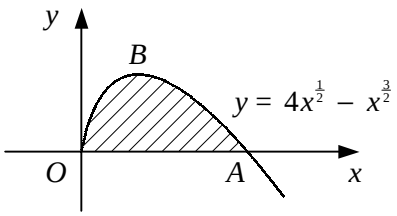
b State, with a reason, whether your answer to part **a** is an under-estimate or an over-estimate of the true area. **(2)**

3 $\mathrm{f}(x) \equiv 3x^{\frac{1}{2}} - x^{-\frac{1}{2}}$.

a Find the value of f(2), giving your answer in the form $k\sqrt{2}$ where k is an exact fraction. **(2)**

b Show that $\int_3^4 \mathrm{f}(x)\,\mathrm{d}x = 12 - 4\sqrt{3}$. **(4)**

4



The diagram shows the curve with the equation $y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$.

The curve meets the x-axis at the origin, O, and at the point A.

a Find the coordinates of the point A. **(2)**

The curve has a maximum at the point B.

b Find the x-coordinate of the point B. **(5)**

c Find the area of the shaded region enclosed by the curve and the x-axis. **(4)**

5 The curve $y = 4 + \frac{1}{x}$ crosses the x-axis at the point $(p, 0)$ and has an asymptote $y = q$.

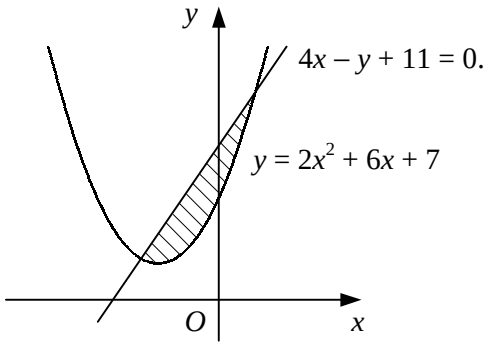
a Write down the values of p and q. **(2)**

b Sketch the curve. **(2)**

The region R is bounded by the curve $y = 4 + \frac{1}{x}$, the x-axis and the lines $x = 1$ and $x = 3$.

c Use the trapezium rule with 5 equally-spaced ordinates to estimate the area of R. **(5)**

6

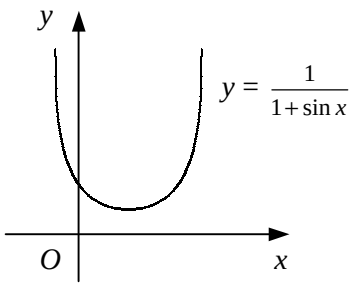


The diagram shows the curve with the equation $y = 2x^2 + 6x + 7$ and the straight line with the equation $4x - y + 11 = 0$.

a Find the coordinates of the points where the curve and line intersect. **(5)**

b Find the area of the shaded region enclosed by the curve and the line. **(6)**

7



The diagram shows the curve with equation $y = \frac{1}{1+\sin x}$, $-\frac{\pi}{2} < x < \frac{3\pi}{2}$.

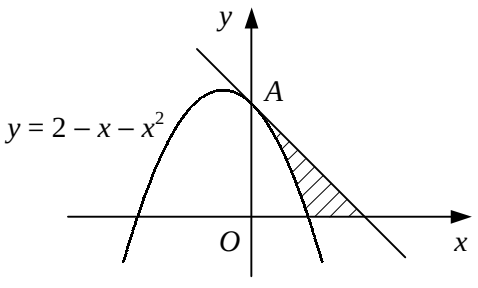
a Find the coordinates of the minimum point of the curve. **(3)**

b Use the trapezium rule with 2 intervals of equal width to estimate the area of the region bounded by the curve, the coordinate axes and the line $x = \frac{\pi}{3}$. **(5)**

8 **a** Expand $(1 + \frac{x}{10})^{12}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient in the expansion. **(4)**

b Using your series expansion from part **a**, find an estimate for $\int_0^1 (1 + \frac{x}{10})^{12}\, dx$. **(5)**

9



The diagram shows the curve with the equation $y = 2 - x - x^2$ and the tangent to the curve at the point A where it crosses the y-axis.

a Find an equation of the tangent to the curve at A. **(4)**

b Show that the area of the shaded region enclosed by the curve, the tangent to the curve at A and the x-axis is $\frac{5}{6}$. **(9)**