INTEGRATION — Answers

1 $= \frac{1}{3}x^3 + 4x^{\frac{3}{2}} - 3x + c$

2 **a** $f(x) = \int (1 - 6x^{-3})\ dx$

$= x + 3x^{-2} + c$

$(1, -2) \Rightarrow -2 = 1 + 3 + c$

$c = -6$

$\therefore f(x) = x - 6 + \frac{3}{x^2}$

b $x = 2 \Rightarrow y = -\frac{13}{4}$, grad $= \frac{1}{4}$

$\therefore$ grad of normal $= -4$

$\therefore y + \frac{13}{4} = -4(x - 2)$

$4y + 13 = -16x + 32$

$16x + 4y - 19 = 0$

3 **a** $f(x) = \int (3x^2 + 2x - 5)\ dx$

$= x^3 + x^2 - 5x + c$

$(3, 22) \Rightarrow 22 = 27 + 9 - 15 + c$

$c = 1$

$\therefore f(x) = x^3 + x^2 - 5x + 1$

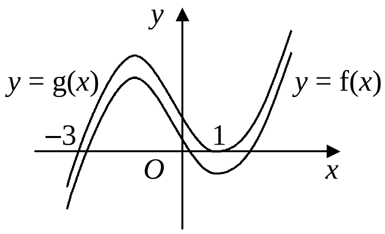
b $g(x) = (x + 3)(x^2 - 2x + 1)$

$= x^3 - 2x^2 + x + 3x^2 - 6x + 3$

$= x^3 + x^2 - 5x + 3$

$= f(x) + 2$

c



4 **a** $= 2x + 6x^{-3}$

b $= \frac{1}{3}x^3 + 3x^{-1} + c$

5 grad of tangent = 12 – 8 – 1 = 3

tangent passes through (0, 0)

$\therefore$ tangent: $y = 3x$

when $x = 2$, $y = 6$

$\therefore$ curve passes through (2, 6)

curve: $y = \int (3x^2 - 4x - 1)\ dx$

$y = x^3 - 2x^2 - x + c$

$(2, 6) \Rightarrow 6 = 8 - 8 - 2 + c$

$c = 8$

$\therefore y = x^3 - 2x^2 - x + 8$

6 **a** $= 3\sqrt{2} - \frac{2}{\sqrt{2}}$

$= 3\sqrt{2} - \sqrt{2}$

$= 2\sqrt{2}$

b $y = \int (3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}})\ dx$

$= 2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + c$

$(4, 7) \Rightarrow 7 = 2(8) - 4(2) + c$

$7 = 16 - 8 + c$

$c = -1$

$\therefore y = 2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} - 1$

when $x = 3$

$y = 6\sqrt{3} - 4\sqrt{3} - 1$

$y = 2\sqrt{3} - 1$

7 **a** $= \int (x^2 + 4x + 4)\ dx$

$= \frac{1}{3}x^3 + 2x^2 + 4x + c$

b $= \int \frac{1}{4}x^{-\frac{1}{2}}\ dx$

$= \frac{1}{2}x^{\frac{1}{2}} + c$

8 **a** $f(x) = \int (3x^2 - 2x - 3)\ dx$

$= x^3 - x^2 - 3x + c$

$(-2, 0) \Rightarrow 0 = -8 - 4 + 6 + c$

$c = 6$

$\therefore f(x) = x^3 - x^2 - 3x + 6$

b $x = 1 \Rightarrow y = 1 - 1 - 3 + 6 = 3$

grad $= 3 - 2 - 3 = -2$

$\therefore y - 3 = -2(x - 1)$

$y - 3 = -2x + 2$

$y = 5 - 2x$

9 **a** $y = \int (2x - 3x^{-2})\ dx$

$= x^2 + 3x^{-1} + c$

$y = 0$ at $x = 1$

$\therefore 0 = 1 + 3 + c$

$c = -4$

$\therefore y = x^2 - 4 + \frac{3}{x}$

b $\frac{d^2y}{dx^2} = 2 + 6x^{-3}$

$\therefore x^2\frac{d^2y}{dx^2} - 2y$

$= x^2(2 + 6x^{-3}) - 2(x^2 - 4 + 3x^{-1})$

$= 2x^2 + 6x^{-1} - 2x^2 + 8 - 6x^{-1}$

$= 8$ [$k = 8$]

10 **a** $= -\frac{1}{2}x^{-2} + c$

b $= \int \frac{x^2 - 2x + 1}{x^{\frac{1}{2}}}\ dx$

$= \int (x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}})\ dx$

$= \frac{2}{5}x^{\frac{5}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

11 **a** $f(x) = \int (4x^3 - 8x)\ dx$

$= x^4 - 4x^2 + c$

$(2, -5) \Rightarrow -5 = 16 - 16 + c$

$c = -5$

$\therefore f(x) = x^4 - 4x^2 - 5$

b $x^4 - 4x^2 - 5 = 0$

$(x^2 + 1)(x^2 - 5) = 0$

$x^2 = -1$ [no sols] or 5

$x = \pm\sqrt{5}$

$\therefore (-\sqrt{5}, 0), (\sqrt{5}, 0)$

12 **a** $y = \int (k - x^{-\frac{1}{2}})\ dx$

$y = kx - 2x^{\frac{1}{2}} + c$

$(1, -2) \Rightarrow -2 = k - 2 + c$

$0 = k + c$ (1)

$(4, 5) \Rightarrow 5 = 4k - 4 + c$

$9 = 4k + c$ (2)

(2) − (1) $9 = 3k$

$k = 3$

b grad $= 3 - 1 = 2$

$\therefore$ grad of normal $= -\frac{1}{2}$

$\therefore y + 2 = -\frac{1}{2}(x - 1)$

$2y + 4 = -x + 1$

$x + 2y + 3 = 0$