INTEGRATION

1 Find

$$\int (x^2 + 6\sqrt{x} - 3)\ \mathrm{d}x.$$ **(3)**

2 The curve $y = \mathrm{f}(x)$ passes through the point (1, –2).

Given that

$$\mathrm{f}'(x) = 1 - \frac{6}{x^3},$$

a find an expression for f(x). **(4)**

The point A on the curve $y = \mathrm{f}(x)$ has x-coordinate 2.

b Show that the normal to the curve $y = \mathrm{f}(x)$ at A has the equation

$$16x + 4y - 19 = 0.$$ **(5)**

3 The curve $y = \mathrm{f}(x)$ passes through the point (3, 22).

Given that

$$\mathrm{f}'(x) = 3x^2 + 2x - 5,$$

a find an expression for f(x). **(4)**

Given also that

$$\mathrm{g}(x) = (x + 3)(x - 1)^2,$$

b show that $\mathrm{g}(x) = \mathrm{f}(x) + 2$, **(3)**

c sketch the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ on the same set of axes. **(3)**

4 Given that

$$y = x^2 - \frac{3}{x^2},$$

find

a $\frac{\mathrm{d}y}{\mathrm{d}x}$, **(2)**

b $\int y\ \mathrm{d}x$. **(3)**

5 The curve C with equation $y = \mathrm{f}(x)$ is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4x - 1.$$

Given that the tangent to the curve at the point P with x-coordinate 2 passes through the origin, find an equation for the curve. **(7)**

6 A curve with equation $y = \mathrm{f}(x)$ is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\sqrt{x} - \frac{2}{\sqrt{x}}, \quad x > 0.$$

a Find the gradient of the curve at the point where $x = 2$, giving your answer in its simplest form. **(2)**

Given also that the curve passes through the point (4, 7),

b find the y-coordinate of the point on the curve where $x = 3$, giving your answer in the form $a\sqrt{3} + b$, where a and b are integers. **(6)**

7 Find

a $\int (x + 2)^2 \; dx$, **(3)**

b $\int \frac{1}{4\sqrt{x}} \; dx$. **(3)**

8 The curve C has the equation $y = f(x)$ and crosses the x-axis at the point P $(-2, 0)$.
Given that

$$f'(x) = 3x^2 - 2x - 3,$$

a find an expression for $f(x)$, **(4)**

b show that the tangent to the curve at the point where $x = 1$ has the equation

$$y = 5 - 2x.$$ **(3)**

9 Given that

$$\frac{dy}{dx} = 2x - \frac{3}{x^2}, \quad x \neq 0,$$

and that $y = 0$ at $x = 1$,

a find an expression for y in terms of x, **(4)**

b show that for all non-zero values of x

$$x^2\frac{d^2y}{dx^2} - 2y = k,$$

where k is a constant to be found. **(4)**

10 Integrate with respect to x

a $\frac{1}{x^3}$, **(2)**

b $\frac{(x-1)^2}{\sqrt{x}}$. **(5)**

11 The curve $y = f(x)$ passes through the point $(2, -5)$.
Given that

$$f'(x) = 4x^3 - 8x,$$

a find an expression for $f(x)$, **(4)**

b find the coordinates of the points where the curve crosses the x-axis. **(4)**

12 The curve C with equation $y = f(x)$ is such that

$$\frac{dy}{dx} = k - x^{-\frac{1}{2}}, \quad x > 0,$$

where k is a constant.

Given that C passes through the points $(1, -2)$ and $(4, 5)$,

a find the value of k, **(5)**

b show that the normal to C at the point $(1, -2)$ has the equation

$$x + 2y + 3 = 0.$$ **(4)**