INTEGRATION — Answers

1 **a** $x^2 + x + c$

b $y = x^2 + x + c$

$(1, 5) \Rightarrow 5 = 1 + 1 + c$

$\therefore c = 3$

$y = x^2 + x + 3$

2 **a** $y = \int (3 - 6x)\, dx$

$y = 3x - 3x^2 + c$

$(2, 1) \Rightarrow 1 = 6 - 12 + c$

$\therefore c = 7$

$y = 3x - 3x^2 + 7$

b $y = \int (3x^2 - x)\, dx$

$y = x^3 - \frac{1}{2}x^2 + c$

$(4, 41) \Rightarrow 41 = 64 - 8 + c$

$\therefore c = -15$

$y = x^3 - \frac{1}{2}x^2 - 15$

c $y = \int (x^2 + 4x + 1)\, dx$

$y = \frac{1}{3}x^3 + 2x^2 + x + c$

$(-3, 4) \Rightarrow 4 = -9 + 18 - 3 + c$

$\therefore c = -2$

$y = \frac{1}{3}x^3 + 2x^2 + x - 2$

d $y = \int (7 - 5x - x^3)\, dx$

$y = 7x - \frac{5}{2}x^2 - \frac{1}{4}x^4 + c$

$(2, 0) \Rightarrow 0 = 14 - 10 - 4 + c$

$\therefore c = 0$

$y = 7x - \frac{5}{2}x^2 - \frac{1}{4}x^4$

e $y = \int (8x - 2x^{-2})\, dx$

$y = 4x^2 + 2x^{-1} + c$

$(\frac{1}{2}, -1) \Rightarrow -1 = 1 + 4 + c$

$\therefore c = -6$

$y = 4x^2 + 2x^{-1} - 6$

f $y = \int (3 - x^{\frac{1}{2}})\, dx$

$y = 3x - \frac{2}{3}x^{\frac{3}{2}} + c$

$(4, 8) \Rightarrow 8 = 12 - \frac{16}{3} + c$

$\therefore c = \frac{4}{3}$

$y = 3x - \frac{2}{3}x^{\frac{3}{2}} + \frac{4}{3}$

3 $f(x) = \int (3 + 2x - x^2)\, dx$

$f(x) = 3x + x^2 - \frac{1}{3}x^3 + c$

$(3, 5) \Rightarrow 5 = 9 + 9 - 9 + c$

$\therefore c = -4$

$f(x) = 3x + x^2 - \frac{1}{3}x^3 - 4$

4 $y = \int (10x^{\frac{3}{2}} - 2x^{-\frac{1}{2}})\, dx$

$y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + c$

$y = 0$ when $x = 7$

$\therefore 7 = 0 + 0 + c$

$c = 7$

$\therefore y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + 7$

when $x = 4$

$y = 4(32) - 4(2) + 7$

$y = 127$

5 **a** $f(x) = \int (2x^3 - x - 8)\ dx$

$f(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 - 8x + c$

$(-1, 4) \Rightarrow 4 = \frac{1}{2} - \frac{1}{2} + 8 + c$

$\therefore c = -4$

$f(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 - 8x - 4$

b at $x = 2$, $y = 8 - 2 - 16 - 4 = -14$

grad $= 16 - 2 - 8 = 6$

$\therefore y + 14 = 6(x - 2)$

$[y = 6x - 26]$

6 $f(x) = \int (3x^2 - 8x - 5)\ dx$

$f(x) = x^3 - 4x^2 - 5x + c$

$(0, 0) \Rightarrow 0 = 0 + c$

$\therefore c = 0$

$f(x) = x^3 - 4x^2 - 5x$

$= x(x^2 - 4x - 5)$

$= x(x + 1)(x - 5)$

crosses x-axis when $f(x) = 0$

$\therefore (-1, 0)$ and $(5, 0)$

7 **a** $y = \int (3x + 2x^{-2})\ dx$

$y = \frac{3}{2}x^2 - 2x^{-1} + c$

b $y = 8$ when $x = 2$

$\therefore 8 = 6 - 1 + c$

$c = 3$

$\therefore y = \frac{3}{2}x^2 - 2x^{-1} + 3$

when $x = \frac{1}{2}$

$y = \frac{3}{8} - 4 + 3$

$y = -\frac{5}{8}$

8 **a** $y = \int (3x^2 + kx)\ dx$

$y = x^3 + \frac{1}{2}kx^2 + c$

$(1, 6) \Rightarrow 6 = 1 + \frac{1}{2}k + c$

$5 = \frac{1}{2}k + c \quad (1)$

$(2, 1) \Rightarrow 1 = 8 + 2k + c$

$-7 = 2k + c \quad (2)$

$(2) - (1) \quad -12 = \frac{3}{2}k$

$k = -8$

b sub. $-7 = -16 + c$

$c = 9$

$\therefore y = x^3 - 4x^2 + 9$